Lesson 10: Equations with Variables on Both Sides

Goal

Example 1
## Steps to Solving Equations

**Equation:**

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Does your equation have fractions?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YES – Multiply EVERY term in the equation (on both sides) by the denominator or the LCD if you have 2 or more fractions.</td>
</tr>
<tr>
<td></td>
<td>NO – Go to step 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2</th>
<th>Does your equation involve the distributive property? (Do you see parenthesis with more than one term inside?)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YES – Rewrite the equation using the distributive property for the terms involved.</td>
</tr>
<tr>
<td></td>
<td>NO – Go to step 3.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3</th>
<th>On either side of the equation, do you have like terms that can be combined?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YES – Rewrite the equation with like terms together. Don’t forget to take the sign in front of the term! Then rewrite the equation again, combining the like terms.</td>
</tr>
<tr>
<td></td>
<td>NO – Go to Step 4.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4</th>
<th>Do you have variables on both sides of the equation?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YES – Add or subtract the terms to get all the variables on one side and all the constants on the other side. Then go to Step 6.</td>
</tr>
<tr>
<td></td>
<td>NO – Go to Step 5.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 5</th>
<th>At this point, you should have a basic two-step equation to solve.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use addition or subtraction to remove any constants from the variable side of the equation.</td>
</tr>
<tr>
<td></td>
<td>Step 6: Use multiplication or division to remove any coefficients from the variable side of the equation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 7</th>
<th>Check your answer</th>
</tr>
</thead>
</table>
Steps to Solving Equations

Equation: ________________________________

Step 1: Does your equation have fractions?

YES – Multiply EVERY term in the equation (on both sides) by the denominator or the LCM if you have 2 or more fractions.

NO – Go to step 2

Step 2: Does your equation involve the distributive property? (Do you see parenthesis with more than one term inside?)

YES – Rewrite the equation using the distributive property for the terms involved.

NO – Go to step 3.

Step 3: On either side of the equation, do you have like terms that can be combined?

YES – Rewrite the equation with like terms together. Don’t forget to take the sign in front of the term! Then rewrite the equation again, combining the like terms.

NO – Go to Step 4.

Step 4: Do you have variables on both sides of the equation?

YES – Add or subtract the terms to get all the variables on one side and all the constants on the other side. Then go to Step 6.

NO – Go to Step 5.

Step 5: At this point, you should have a basic two-step equation to solve.

Use addition or subtraction to remove any constants from the variable side of the equation.

Step 6: Use multiplication or division to remove any coefficients from the variable side of the equation.

Step 7: Check your answer
### Steps to Solving Equations - Example
#### Equations with Variables on Both Sides

Use the **7 Solving Equations Steps** to solve the following equation:

\[
\frac{2}{3}(x+3) + 2 = 3x + \frac{1}{3}(3x - 4)
\]

<table>
<thead>
<tr>
<th>Step 1: Does your equation have fractions?</th>
<th>YES: Remove the fraction by multiplying every term by 3 (the denominator). I highlighted each term a different color. Each term was multiplied by 3.</th>
</tr>
</thead>
</table>
| YES – Multiply EVERY term in the equation (on both sides) by the denominator. | \[
\frac{2}{3}(x+3) + 2 = 3x + \frac{1}{3}(3x - 4) \\
2(x+3) + 6 = 9x + 1(3x-4)
\] |
| NO – Go to step 2 |  |

<table>
<thead>
<tr>
<th>Step 2: Does your equation involve the distributive property? (Do you see parenthesis with more than one term inside?)</th>
<th>YES: Distribute.</th>
</tr>
</thead>
</table>
| YES – Rewrite the equation using the distributive property for the terms involved. | \[
2(x+3) + 6 = 9x + 1(3x-4) \\
2x +6 +6 = 9x + 3x -4
\] |
| NO – Go to step 3 |  |

<table>
<thead>
<tr>
<th>Step 3: On either side of the equation, do you have like terms that can be combined?</th>
<th>YES: Combine like terms!</th>
</tr>
</thead>
</table>
| YES – Rewrite the equation with like terms together. Don’t forget to take the sign in front of the term! Then rewrite the equation again, combining the like terms. | \[
2x +6 +6 = 9x + 3x -4 \\
2x +12 = 12x - 4
\] |
| NO – Go to Step 4 |  |

<table>
<thead>
<tr>
<th>Step 4: Do you have variables on both sides of the equation?</th>
<th>YES</th>
</tr>
</thead>
</table>
| YES – Add or subtract the terms to get all the variables on one side and all the constants on the other side. Then go to Step 6. | \[
2x +12 = 12x - 4 \\
2x – 2x + 12 = 12x – 2x - 4 \\
12 = 10x -4 \\
12 + 4 = 10x -4 +4 \\
16 = 10x
\] |
| NO – Go to Step 5 |  |

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<tr>
<th>Step 5: At this point, you should have a basic two-step equation to solve.</th>
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<td>Use addition or subtraction to remove any constants from the variable side of the equation.</td>
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<tr>
<th>Step 6: Use multiplication or division to remove any coefficients from the variable side of the equation.</th>
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</thead>
</table>
| \[
16 = 10x \\
10 = 10 \\
8/5 = x
\] |  |

<table>
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<tr>
<th>Step 7: Check your answer</th>
<th></th>
</tr>
</thead>
</table>
| \[
\frac{2}{3}(x+3) + 2 = 3x + \frac{1}{3}(3x - 4) \\
\frac{2}{3}(\frac{8}{5}+3) + 2 = 3x + \frac{1}{3}(3\cdot\frac{8}{5} -4) \\
5.06666667 = 5.06666667
\] |  |
Lesson 10: Solving Equations with Variables on Both Sides

Directions: Solve each problem on your own paper. Use the Steps to Solving Equations organizer if needed.

1. $3x + 2 = 2x - 5$
2. $2(x + 3) = 5x - 2$
3. $3(x + 3) = -2(x - 4)$
4. $-2x + 3(x - 2) = -4(2x - 2)$
5. $-5x + 4(x - 2) = 3 - (3 - 2x)$
6. $\frac{1}{3}x - 4 = 9x - \frac{3}{3}$
7. $\frac{1}{4}y - 2 = 3 - \frac{3}{4}y$
8. $\frac{1}{2}(x - 5) = 3(2x - 4)$
9. $2y + \frac{1}{2}(y - 5) = \frac{1}{2}(y + 3) - 4$
10. $3x - 2(x - 4) = -2(x + 3)$
11. $6y - (y - 4) = 2(2y + 3)$
12. $2x + 3(x - 5) = 2(2x - 6) + 2x - 4$

13. Each side of the triangle has the same length. What is the perimeter of the triangle?

14. Find the perimeter of the rectangle.

Your next assignment will be Quiz #2. This quiz will assess the following skills:

- Distributive Property and Equations
- Equations with variables on both sides
- Equations with Fractions
- Literal Equations

Complete the study guide first.
Directions: For numbers 1-3, solve each equation. Show every step. (2 points each)

1. $-2x + 4 = -4(x-6)$

2. $\frac{1}{3}x - 6 = -x - \frac{2}{3}(6-x)$

3. $2x + 6(x - 5) = 2(2x - 15) + 3x + 6$

4. Each side of the Pentagon is equal in length. Find the perimeter of the pentagon. (2 points)

The perimeter is $3r + 5 + 5r - 3 = 8r + 2$. Therefore, the perimeter of the pentagon is $8r + 2$. 

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Lesson 10: Solving Equations with Variables on Both Sides

Answer Key

1. \(3x + 2 = 2x - 5\)
   \[3x + 2 - 2 = 2x - 5 - 2\]
   \[3x = 2x - 7\]
   \[3x - 2x = 2x - 2x - 7\]
   \[x = -7\]

2. \(2(x + 3) = 5x - 2\)
   \[2x + 6 = 5x - 2\]
   \[2x + 6 - 6 = 5x - 2 - 6\]
   \[2x = 5x - 8\]
   \[2x - 5x = 5x - 5x - 8\]
   \[-3x = -8\]
   \[-3\]
   \[x = 8/3\]

3. \(3(x + 3) = -2(x - 4)\)
   \[3x + 9 = -2x + 8\]
   \[3x + 9 - 9 = -2x + 8 - 9\]
   \[3x = -2x - 1\]
   \[3x + 2x = -2x + 2x - 1\]
   \[5x = -1\]
   \[5 \quad 5\]
   \[x = -1/5\]

4. \(-2x + 3(x - 2) = -4(2x - 2)\)
   \[-2x + 3x - 6 = -8x + 8\]
   \[x - 6 = -8x + 8\]
   \[x + 8x - 6 = -8x + 8x + 8\]
   \[9x - 6 = 8\]
   \[9x - 6 + 6 = 8 + 6\]
   \[9x = 14\]
   \[9 \quad 9\]
   \[x = 14/9\]
5. \(-5x + 4(x - 2) = 3 - (3-2x)\)
\[-5x + 4x - 8 = 3 - 3 + 2x\]
\[-x - 8 = 2x\]
\[-x + x - 8 = 2x + x\]
\[-8 = 3x\]
\[\frac{-8}{3} = x\]

6. \(\frac{1}{3}x - 4 = 9x - \frac{2}{3}\)
\[3\left[\frac{1}{3}x - 4\right] = [9x - \frac{2}{3}]3\]
\[1x - 12 = 27x - 2\]
\[1x - 1x - 12 = 27x - 1x - 2\]
\[-12 = 26x - 2\]
\[-12 + 2 = 26x - 2 + 2\]
\[\frac{-10}{26} = 26x\]
\[\frac{-10}{26} = \frac{26x}{26}\]
\[\frac{-5}{13} = x\]

7. \(\frac{1}{4}y - 2 = 3 - \frac{3}{4}y\)
\[4\left[\frac{1}{4}y - 2\right] = [3 - \frac{3}{4}y]4\]
\[1y - 8 = 12 - 3y\]
\[1y + 3y - 8 = 12 - 3y + 3y\]
\[4y - 8 = 12\]
\[4y - 8 + 8 = 12 + 8\]
\[4y = 20\]
\[\frac{4y}{4} = \frac{20}{4}\]
\[y = 5\]

8. \(\frac{1}{2}(x - 5) = 3(2x - 4)\)
\[2\left[\frac{1}{2}(x - 5)\right] = [3(2x - 4)]2\]
\[1(x - 5) = 6(2x - 4)\]
\[1x - 5 = 12x - 24\]
\[1x - 1x - 5 = 12x - 1x - 24\]
\[-5 = 11x - 24\]
\[-5 + 24 = 11x - 24 + 24\]
\[19 = 11x\]
\[\frac{19}{11} = \frac{11x}{11}\]
\[\frac{19}{11} = x\]
9. \[2y + \frac{1}{2}(y - 5) = \frac{1}{2}(y + 3) - 4\]

\[2[2y + \frac{1}{2}(y - 5)] = [\frac{1}{2}(y + 3) - 4]2\]

\[4y + 1(y - 5) = 1(y + 3) - 8\]

\[4y + y - 5 = y + 3 - 8\]

\[5y - 5 = y - 5\]

\[5y - 5 + 5 = y - 5 + 5\]

\[5y = y + 0\]

\[5y - y = y - y\]

\[4y = 0\]

\[\frac{4y}{4} = \frac{0}{4}\]

\[y = 0\]

10. \[3x - 2(x - 4) = -2(x + 3)\]

\[3x - 2x + 8 = -2x - 6\]

\[6x - 2x + 8 = -2x - 6\]

\[4x + 8 = -2x - 6\]

\[x + 8 = -2x - 6\]

\[x + 8 - 8 = -2x - 6 - 8\]

\[x = -2x - 14\]

\[x + 2x = -2x + 2x - 14\]

\[3x = -14\]

\[\frac{3x}{3} = \frac{-14}{3}\]

\[x = -\frac{14}{3}\]

11. \[6y - (y - 4) = 2(2y + 3)\]

\[6y - y + 4 = 4y + 6\]

\[5y + 4 = 4y + 6\]

\[5y + 4 - 4 = 4y + 6 - 4\]

\[5y = 4y + 2\]

\[5y - 4y = 4y - 4y + 2\]

\[y = 2\]

12. \[2x + 3(x - 5) = 2(2x - 6) + 2x - 4\]

\[2x + 3x - 15 = 4x - 12 + 2x - 4\]

\[5x - 15 = 4x + 2x - 12 - 4\]

\[5x - 15 = 6x - 16\]

\[5x - 15 + 15 = 6x - 16 + 15\]

\[5x = 6x - 1\]

\[5x - 6x = 6x - 6x - 1\]

\[-x = -1\]

\[\frac{-x}{-1} = \frac{-1}{-1}\]

\[x = 1\]
13. Each side of the triangle has the same length. What is the perimeter of the triangle?

If each side of the triangle has the same length, then we can set the two sides of this triangle equal to each other in order to find the value of s. Once we find the value of s, we can substitute for one side of the triangle to find the length of the sides.

\[
\begin{align*}
4s + 5 &= s + 11 \\
4s - s + 5 &= s - s + 11 \\
3s + 5 &= 11 \\
3s + 5 - 5 &= 11 - 5 \\
3s &= 6 \\
3s/3 &= 6/3 \\
s &= 2
\end{align*}
\]

s = 2 Now let’s substitute to find the length of 1 side.

\[
\begin{align*}
s + 11 \\
2 + 11 &= 13
\end{align*}
\]

The length of one side is 13. There are three sides, all the same length. So,

\[
13(3) = 39
\]

The perimeter of the triangle is 39.

14. Find the perimeter of the rectangle.

The lengths of a rectangle are the same dimensions. Therefore, we can set the two lengths equal to each other to solve for y.

Once we find the value of y, we can substitute to find the length of the rectangle.

\[
\begin{align*}
7y &= y + 18 \\
7y - y &= y - y + 18 \\
6y &= 18 \\
6y/6 &= 18/6 \\
y &= 3
\end{align*}
\]

Y = 3 Now let’s substitute to find the length of the rectangle.

\[
\begin{align*}
7y \\
7(3) &= 21 \\
The length of the rectangle is 21. The width is given: 6 \\
P &= 2l + 2w \\
P &= 2(21) + 2(6) \\
P &= 42 + 12 \\
P &= 54 \\
The perimeter of the rectangle is 54.
\end{align*}
\]
Directions: For numbers 1-3, solve each equation. Show every step. (2 points each)

1. \(-2x + 4 = -4(x-6)\)

\[-2x + 4 = -4x + 24\]  Distribute the -4 throughout the parenthesis

\[-2x + 4 - 4 = -4x + 24 - 4\]  Subtract 4 from both sides

\[-2x = -4x + 20\]  Simplify: \(24-4 = 20\)

\[-2x + 4x = -4x + 4x + 20\]  Add 4x to both sides

\[2x = 20\]  Simplify: \(-2x + 4x = 20\)

\[2x/2 = 20/2\]  Divide by 2 on both sides

\[x = 10\]  Simplify: \(20/2 = 10\)

2. \(\frac{1}{3}x - 6 = -x - \frac{2}{3}(6-x)\)

\[3[\frac{1}{3}x - 6] = 3[-x - \frac{2}{3}(6-x)]\]  Multiply by 3 on both sides to get rid of fraction

\[x - 18 = -3x - 2(6-x)\]  Simplify

\[x - 18 = -3x - 12 + 2x\]  Distribute -2 throughout the parenthesis

\[x - 18 = -3x + 2x - 12\]  Rewrite like terms together

\[x - 18 = -x - 12\]  Combine like terms: \(-3x + 2x = -x\)

\[x - 18 + 18 = -x - 12 + 18\]  Add 18 to both sides

\[x = -x + 6\]  Simplify: \(-12 + 18 = 6\)

\[x + x = -x + x + 6\]  Add x to both sides

\[2x = 6\]  Simplify: \(x + x = 2x\)

\[2x/2 = 6/2\]  Divide by 2 on both sides

\[x = 3\]  Simplify: \(6/2 = 3\)
3. \[ 2x + 6(x - 5) = 2(2x - 15) + 3x + 6 \]

\[
\begin{align*}
2x + 6x - 30 &= 4x - 30 + 3x + 6 \\
2x + 6x - 30 &= 4x + 3x - 30 + 6 \\
8x - 30 &= 7x - 24 \\
8x - 30 + 30 &= 7x - 24 + 30 \\
8x &= 7x + 6 \\
8x - 7x &= 7x - 7x + 6 \\
x &= 6
\end{align*}
\]

Distribute the 6 and 2 throughout the parenthesis
Rewrite with like terms together
Combine like terms
Add 30 to both sides
Simplify: \(-24 + 30 = 6\)
Subtract 7x from both sides
Simplify: \(8x - 7x = x\)

4. Each side of the Pentagon is equal in length. Find the perimeter of the pentagon. (2 points)

Since each side of the pentagon is equal in length, we can set the sides that we know equal to each other and solve for the variable \(r\). Once we find the value of \(r\), we can find the value of one side of the pentagon and then multiply by 5 to get the perimeter.

\[
\begin{align*}
3r + 5 &= 5r - 3 \\
3r - 3r + 5 &= 5r - 3r - 3 \\
5 &= 2r - 3 \\
5 + 3 &= 2r - 3 + 3 \\
8 &= 2r \\
8/2 &= 2r/2 \\
4 &= r
\end{align*}
\]

Set the 2 sides equal to each other
Subtract 3r from both sides
Simplify: \(5r - 3r = 2r\)
Add 3 to both sides
Simplify: \(5 + 3 = 8\)
Divide by 2 on both sides
Simplify: \(8/2 = 4\)

Since \(r = 4\), we can substitute 4 for \(r\).

\[3r + 5 = 3(4) + 5 = 17\]

One side is equal to 17 units, so the perimeter is \(17(5) = 85\) units. (Multiply by 5 because the pentagon has 5 sides)

The perimeter of the pentagon is 85 units.