Lesson 1: Graphing Systems of Equations

A system of equations is:

Graph the system of equations and identify the solution.

\[ y = \frac{1}{2}x - 4 \quad \& \quad y = -\frac{5}{2}x + 2 \]

The solution to a system of equations is ______________________________________________________

________________________________________________________________________________________

The solution is ____________________________________________________________
Example 2

Graph the system of equations and Identify the solution.
\[ y = \frac{3}{4}x - 1 \quad \text{and} \quad 3x - 4y = -16 \]

Parallel Lines

Example 3

Graph the system of equations and Identify the solution.
\[ y = 3x + 6 \quad \text{and} \quad 6x - 2y = -12 \]

Equivalent Lines (Same Line)
Points to Remember!

- When you graph two or more equations, the **point of intersection** is the **solution** to the system.

  The point of intersection indicates that this solution is the same for both equations.

  It is the only time that both equations will have exactly the same solution.

- When two or more equations have the **same slope**, the lines will be **parallel**.

  If the lines are **parallel**, the system of equations **do not have a solution**.
• When two or more equations have the **same slope and same y-intercept**, then the lines are exactly the **same** line. (They lie on top of each other.)

Therefore, there are an **infinite** number of solutions!

**Every point** on the lines is a solution to the system of equations.

Both equations have a slope of 1 and a y-intercept of 5.
Lesson 1: Graphing Systems of Equations

1. Which ordered pair is the solution to the System of Equations?
   A. (2,3)  
   B. (3,2)  
   C. (-2,3)  
   D. (-3,-3) 

2. Which answer best describes the solution to the system of equations?
   A. (1, 1)  
   B. Infinite solutions  
   C. No solution  
   D. None of the above.
3. Graph the following system of equations and identify the solution.

\[
y = -3x + 6 \\
y = \frac{1}{3}x - 4
\]

4. Graph the following system of equations and identify the solution.

\[
y = \frac{1}{2}x + 3 \\
x + 2y = 10
\]
5. Graph the following system of equations and identify the solution.

\[ y = -\frac{2}{3}x + 2 \]
\[ 2x + 3y = 6 \]

6. Graph the following system of equations and identify the solution.

\[ y = -2x + 4 \]
\[ 2x + y = 6 \]
7. John graphed the following system of equations:
   \[ y = 8x - 9 \]
   \[ y = 8x + 2 \]

Which of these statements best describes the relationship between the two lines?

A. They have one point in common.
B. They have no points in common.
C. They have two points in common.
D. They have infinite number of points in common.

8. Estimate the solution to the following system of equations.

   A. \((-1, 5)\)
   B. \((-1.5, 5)\)
   C. \((1, 5)\)
   D. \((1.5, 5)\)

9. Describe the relationship between the following two lines. Explain your reasoning.

   \[ y = 5x - 9 \]
   \[ y = 5x + 10 \]
10. Explain what the solution to a system of equations means.

11. U Drive Cab Co. charges $2 per mile in addition to a $2 flat rate. Yellow Taxi Co. charges a $5 base fee plus $1 per mile. Write a linear system of equations that represents this situation.
   • Graph the system of equations.
   • After how many miles will the two companies charge the same amount?

   Let x = ____________________  Let y = ______________ ______
   U Drive Co.      Yellow Taxi Co.
   
   y = mx + b       y = mx +b

   _________________   _________________  _________________   _________________

   y = mx + b       y = mx +b

   __________________
12. Stickers.com charges $0.50 per sticker in addition to a $4 shipping fee. Stickums.com charges $2 for shipping and $0.75 a sticker. Write a linear system of equations that represents this situation.

- Graph the system of equations.
- How many stickers would you need to buy in order for the two companies to charge exactly the same amount?
- If you needed to buy 15 stickers, which company would cost the least? Justify your answer.

Let \( x = \) ____________________  Let \( y = \) ____________________

Stickers.com

<table>
<thead>
<tr>
<th>Slope (m)</th>
<th>Y-intercept (b)</th>
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\[ y = mx + b \]

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Stickums.com

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\[ y = mx + b \]

__________
13. You can download music from Musicrocks.com for $0.50 a song if you pay the one-time $5 set up fee.
   You can download music from Areyoulistening.com for $1.00 with no set up fee.
   - Write a system of equations to represent this situation.
   - Graph the linear system of equations.
   - You need to download 10 songs. Which company would you choose? Why?

Let \( x = \) ____________________  
Let \( y = \) ____________________

Musicrocks.com

<table>
<thead>
<tr>
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\[ y = mx + b \]

Areyoulistening.com

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\[ y = mx + b \]
Lesson 1: Graphing Systems of Equations – Answer Key

1. Which ordered pair is the solution to the System of Equations?

   A. (2,3)
   B. (3,2)
   C. (-2,3)
   D. (-3,-3)

   ![Graph with solution point (-2,3)]

2. Which answer best describes the solution to the system of equations?

   A. (1, 1)
   B. Infinite solutions
   C. No solution
   D. None of the above.

   ![Graph showing parallel lines]

When two lines are parallel, the system of equations has no solution. The lines never intersect, therefore there is no solution.
3. Graph the following system of equations and identify the solution.

\[ y = -3x + 6 \]
\[ y = \frac{1}{3}x - 4 \]

These equations are pretty easy to graph because they are both written in slope intercept form. Graph the y-intercept and use the slope to find your next point.

The solution is the point of intersection for the two lines. The point of intersection is \((3,-3)\).

**The solution to this system of equations is:**

\((3,-3)\)

4. Graph the following system of equations and identify the solution.

\[ y = \frac{1}{2}x + 3 \]
\[ x + 2y = 10 \]

The first equation is written in slope intercept form, so that is easy to graph.

The second equation is in standard form. This equation is set up so that it’s easy to find the x and y intercept, so I am going to use that method to graph the line. You can also rewrite the equation in slope intercept form.

\[ x + 2y = 10 \]

Let \(x = 0\)  
Let \(y = 0\)

\[ 0 + 2y = 10 \]
\[ x + 2(0) = 10 \]

\[ 2y = 10 \]
\[ 2 \]
\[ y = 5 \]

The solution to this system of equations is \((2,4)\). That is the point of intersection.
5. Graph the following system of equations and identify the solution.

\[ y = \frac{-2}{3}x + 2 \]
\[ 2x + 3y = 6 \]

The first equation \( y = \frac{-2}{3}x + 2 \) is written in slope intercept form. That is easy to graph.

The second equation is in standard form. I can either find the x and y intercepts or rewrite in slope intercept form. I think that finding the intercepts is easier.

\[ 2x + 3y = 6 \]
Let \( x = 0 \) \hspace{1em} Let \( y = 0 \)

\[ 2(0) + 3y = 6 \hspace{1em} 2x + 3(0) = 6 \]
\[ 3y = 6 \hspace{1em} 2x = 6 \]
\[ y = 2 \hspace{1em} x = 3 \]

Notice that the red line sits on top of the blue line. These equations are equivalent, therefore, they graph the same line. There are an infinite number of solutions! Every point on the line is a solution to the system of equations!

6. Graph the following system of equations and identify the solution.

\[ y = -2x + 4 \]
\[ 2x + y = 6 \]

The first equation \( y = -2x + 4 \) is written in slope intercept form. This is easy to graph!

The second equation is in standard form. This time, I am going to rewrite the equation in slope intercept form. You could also find the intercepts!

\[ 2x + y = 6 \]
\[ 2x - 2x + y = 6 - 2x \]
\[ y = -2x + 6 \]

When written in slope intercept form, notice that the two equations have the same slope. That is an indication that the lines will be parallel. Parallel lines have no solution!
7. John graphed the following system of equations:
   \[ y = 8x - 9 \]
   \[ y = 8x + 2 \]

   Which of these statements best describes the relationship between the two lines?

   A. They have one point in common.
   B. They have no points in common.
   C. They have two points in common.
   D. They have infinite number of points in common.

   Notice in the two equations that the slope is the same (8) and the y-intercepts are different. When two equations have the same slope with different y-intercepts they will always be parallel to each other. Parallel lines never intersect; therefore, they have no points in common and no solution!

8. Estimate the solution to the following system of equations.

   A. (-1,5)
   B. (-1.5,5)
   C. (1,5)
   D. (1.5,5)

   The solution is between -1 and -2 on the x axis. We can estimate it to be about -1.5 on the x axis. The point of intersection is (-1.5,5)

9. Describe the relationship between the following two lines. Explain your reasoning.

   \[ y = 5x - 9 \]
   \[ y = 5x + 10 \]

   The two lines in this system of equation are parallel lines. I know they are parallel because they have the same slope but different y-intercepts. The lines will never intersect because they are parallel. This system of equations does not have a solution.
10. Explain what the solution to a system of equations means.

The solution to a system of equations is the point where the 2 lines intersect. The point of intersection is the ONLY point that is a solution to BOTH equations. It is the only point that the 2 equations share. This is the only time when both equations have exactly the same solution!

11. U Drive Cab Co. charges $2 per mile in addition to a $2 flat rate. Yellow Taxi Co. charges a $5 base fee plus $1 per mile. Write a linear system of equations that represents this situation.
   - Graph the system of equations.
   - After how many miles will the two companies charge the same amount?

Let \( x = \text{The number of miles driven} \) 
Let \( y = \text{The total cost} \)

U Drive Co.  
\[
y = mx + b \\
y = 2x + 2
\]

Yellow Taxi Co.  
\[
y = mx + b \\
y = x + 5
\]

The two companies will charge the same amount after 3 miles. They will both charge $8.00.
12. Stickers.com charges $0.50 per sticker in addition to a $4 shipping fee. Stickums.com charges $2 for shipping and $0.75 a sticker. Write a linear system of equations that represents this situation.

- Graph the system of equations.
- How many stickers would you need to buy in order for the two companies to charge exactly the same amount?
- If you needed to buy 15 stickers, which company would cost the least? Justify your answer.

Let \( x \) = **The number of stickers**

Let \( y \) = **The total cost**

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<thead>
<tr>
<th>Slope (m)</th>
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<tr>
<td>0.50</td>
<td>4</td>
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</table>

\[ y = 0.50x + 4 \]

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<tr>
<th>Slope (m)</th>
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<tbody>
<tr>
<td>0.75</td>
<td>2</td>
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</table>

\[ y = 0.75x + 2 \]

**NOTE: Rewrite decimals as fractions for graphing purposes**

\[ y = \frac{1}{2}x + 4 \]

\[ y = \frac{3}{4}x + 2 \]

In order for the two companies to charge the same amount you would need to buy 8 stickers.

If you were to buy 15 stickers, **Stickers.com** would cost the least. According to the graph, after the point of intersection (8,8), Stickers.com is cheaper.

**Justify:**

\[
\text{Stickers.com: } y = 0.5(15) + 4 \\
\text{ } \ \ Y = 11.5 \\
\text{Stickums.com: } y = 0.75(15) + 2 \\
\text{ } \ \ Y = 13.25 \\
\text{Stickers.com is cheaper!}
\]
13. You can download music from Musicrocks.com for $0.50 a song if you pay the one-time $5 set up fee.
You can download music from Areyoulistening.com for $1.00 with no set up fee.

- Write a system of equations to represent this situation.
- Graph the linear system of equations.
- You need to download 10 songs. Which company would you choose? Why?

Let \( x \) = the number of songs  
Let \( y \) = The total cost

Musicrocks.com

\[
\begin{array}{c|c}
\text{Slope (m)} & \text{Y-intercept (b)} \\
0.50 & 5 \\
\end{array}
\]

\[ y = mx + b \]

\[ y = 0.50x + 5 \]  
(y = \frac{1}{2}x +5)

Areyoulistening.com

\[
\begin{array}{c|c}
\text{Slope (m)} & \text{Y-intercept (b)} \\
1 & 0 \\
\end{array}
\]

\[ y = mx + b \]

\[ y = x \]

If I needed to download 10 songs, it wouldn’t matter which company I chose, because they would cost the same amount. The point of intersection is \((10,10)\). For 10 songs, both companies would charge $10.
Lesson 2: Using the Substitution Method

Steps for Using the Substitution Method

1. __________________________________________________________________________

2. __________________________________________________________________________

3. __________________________________________________________________________

4. __________________________________________________________________________

Example 1

Solve the system of equations using substitution.

\[ y = x + 3 \quad \& \quad y = -\frac{3}{2}x - 2 \]
Example 2

Solve the system of equations using substitution.

\[3x + y = 2 \quad \& \quad 6x + 2y = -8\]

Example 3

Solve the system of equations using substitution.

\[2x + y = 1 \quad \& \quad 6x + 3y = 3\]
Lesson 2: Using the Substitution Method

1. Which ordered pair is the solution to the following system of equations?
   \[ y = x + 1 \]
   \[ 2x + y = -2 \]
   A. (-1,0)  
   B. (1,-2)  
   C. Infinite possibilities  
   D. (-2,1)

2. Which ordered pair is the solution to the following system of equations?
   \[ x - 2y = 3 \]
   \[ 3x + 6y = -3 \]
   A. (-1,1)  
   B. (3,-3)  
   C. (1,-1)  
   D. (-3,3)

3. Solve the following systems of equations using substitution.
   \[ 2x + 2y = 3 \]
   \[ x - 4y = -1 \]

4. Solve the following systems of equations using substitution. Explain your answer.
   \[ y = -3x + 10 \]
   \[ 3x + y = 5 \]
5. Find the solution to the system of equations. Explain your answer.

\[2x + y = 2\]
\[8x + 4y = 8\]

6. Describe the graph for the following system of equations:

\[4y = 8x + 8\]
\[-2x + y = 7\]

7. Find the solution to the system of equations and describe the graph.

\[x = 2y + 1\]
\[3x + y = 10\]

8. Which ordered pair is the solution to the system of equations?

\[y = 4x + 2\]
\[-8x + 2y = 8\]

A. (2,8)  
B. (4,8)  
C. (0,0)  
D. None of the Above
9. Find the solution to the system using substitution. Then check your answer by graphing.
   \[
   \begin{align*}
   y &= 2x - 8 \\
   x + y &= 7
   \end{align*}
   \]

10. Which answer best describes the solution to the following system of equations?
    \[
    \begin{align*}
    y &= 4x - 2 \\
    y &= 3x + 3
    \end{align*}
    \]

   A. (5, 18)
   B. (18, 5)
   C. (-2, 3)
   D. None of the above

11. Solve the following system of equations using substitution.
    \[
    \begin{align*}
    y &= \frac{3}{4}x - 6 \\
    y &= \frac{1}{2}x - 4
    \end{align*}
    \]
12. You are buying pizzas for a luncheon. Pizza Palace charges $5.50 a pizza and a $10 delivery and set up fee. Jerry’s Pizza charges $6.00 a pizza and a $7 delivery and set up fee.

• Write a linear system of equations to represent this situation.

• How many pizzas would you need to buy in order for the two companies to charge the same amount? How much would the pizzas cost?

• If you needed to order 10 pizzas, which company would be the best option in terms of cost? Explain your answer.

Your next assignment will be a quiz on the following concepts:

• Graphing a System of Equations.
• Solving a system of equations using the substitution method.

1. Solve the system of equations using the substitution method. Justify your answer mathematically. (3 points)

   \[ Y = \frac{1}{3}x + 2 \]
   \[ Y = 3x - 5 \]

2. Solve the system of equations using the substitution method. If you were to graph this system of equations, describe what the graph would look like. (3 points)

   \[ Y = -\frac{2}{3}x + 6 \]
   \[ 2x + 3y = -12 \]

3. Hannah has $310 and saves $8 per week. Kara has $220 and saves $12 per week. After how many weeks will Hannah and Kara have the same amount of money? (4 points)

   • Write a system of equations to describe this situation.
   • Solve the system of equations using the substitution method.
   • How much money will Hannah and Kara have when their savings are the same?
Lesson 2: Using the Substitution Method – Answer Key

1. Which ordered pair is the solution to the following system of equations?
   \[ \begin{align*}
   y &= x + 1 \\
   2x + y &= -2
   \end{align*} \]
   
   A. (-1,0)  
   B. (1,-2)  
   C. Infinite possibilities  
   D. (-2,1)

   Since the first equation is written as \( y = x + 1 \), it is ready to be substituted for \( y \) into the second equation.

   \[ \begin{align*}
   2x + y &= -2 \\
   2x + x + 1 &= -2 \\
   3x + 1 &= -2 \\
   3x + 1 - 1 &= -2 - 1 \\
   3x &= -3 \\
   x &= -1 \\
   \end{align*} \]

   The first equation is written as \( y = x + 1 \), it is ready to be substituted for \( y \) into the second equation.

   \[ \begin{align*}
   2x + y &= -2 \\
   2x + x + 1 &= -2 \\
   3x + 1 &= -2 \\
   3x + 1 - 1 &= -2 - 1 \\
   3x &= -3 \\
   y &= x + 1 \\
   3x &= -3 \\
   3x &= -3 \\
   x &= -1 \\
   y &= -1 + 1 \\
   y &= 0 \\
   \end{align*} \]

   Ordered Pair: \((-1,0)\)

2. Which ordered pair is the solution to the following system of equations?
   \[ \begin{align*}
   x - 2y &= 3 \\
   3x + 6y &= -3
   \end{align*} \]
   
   A. (-1,1)  
   B. (3,-3)  
   C. (1,-1)  
   D. (-3,3)

   The first equation is easiest to rewrite. I can solve this equation for \( x \).

   \[ \begin{align*}
   x - 2y + 2y &= 3 + 2y \\
   x &= 2y + 3 \\
   \end{align*} \]

   Substitute \( 2y + 3 \) for \( x \) into the second equation:

   \[ \begin{align*}
   3x + 6y &= -3 \\
   3(2y + 3) + 6y &= -3 \\
   6y + 9 + 6y &= -3 \\
   12y + 9 &= -3 \\
   12y + 9 - 9 &= -3 - 9 \\
   12y &= -12 \\
   y &= -1 \\
   \end{align*} \]

   Solution: \((1,-1)\)
3. Solve the following systems of equations using substitution.

\[
\begin{align*}
2x + 2y &= 3 \\
x - 4y &= -1
\end{align*}
\]

The second equation is easiest to rewrite. Since the x term does not have a coefficient, it is easiest to solve for x.

\[
x - 4y = -1
\]

\[
x - 4y + 4y = -1 + 4y \quad \text{Add 4y to both sides}
\]

\[
x = 4y - 1
\]

Substitute 4y - 1 for x into the first equation.

\[
2x + 2y = 3
\]

\[
2(4y - 1) + 2y = 3 \quad \text{Substitute 4y-1 for x.}
\]

\[
8y - 2 + 2y = 3 \quad \text{Distribute 2}
\]

\[
10y - 2 = 3 \quad \text{Combine like terms (8y + 2y = 10y)}
\]

\[
10y - 2 + 2 = 3 + 2 \quad \text{Add 2 to both sides.}
\]

\[
10y = 5 \\
\frac{10y}{10} = \frac{5}{10}
\]

\[
y = \frac{1}{2}
\]

The solution to this system of equations is \((1, \frac{1}{2})\).

4. Solve the following systems of equations using substitution. Explain your answer.

\[
y = -3x + 10
\]

\[
3x + y = 5
\]

The first equation is already written in slope intercept form. Therefore, it will be easy to substitute -3x + 10 for y into the second equation.

\[
3x + y = 5
\]

\[
3x + (-3x + 10) = 5 \quad \text{Substitute -3x +10 for y.}
\]

\[
3x - 3x + 10 = 5 \quad 3x - 3x = 0
\]

\[
10 = 5 \quad \text{I’m left with 10 = 5}
\]

**When my end result is a statement that does not make sense, like 10 = 5, then I know that there is no solution to this system of equations. If there is no solution, then the lines must be parallel.**

Note: If you want to check, rewrite the second equation in slope intercept form. You will get y = -3x +5. Notice that both equations have a slope of -3!
5. Find the solution to the system of equations. Explain your answer.

\[
\begin{align*}
2x + y & = 2 \\
8x + 4y & = 8
\end{align*}
\]

**Step 1:** I will need to rewrite one of the equations as \( y = \ldots \) or \( x = \ldots \). The first equation is easiest to rewrite as \( y = \ldots \).

\[
2x + y = 2
\]

\[
2x - 2x + y = 2 - 2x
\]

Subtract 2x from both sides.

\[
y = -2x + 2
\]

**Step 2:** Substitute \(-2x + 2\) for \( y \) into the second equation.

\[
\begin{align*}
8x + 4y &= 8 \\
8x + 4(-2x + 2) &= 8 & \text{Substitute } -2x + 2 \text{ for } y \\
8x - 8x + 8 &= 8 & \text{Distribute 4.} \\
8 &= 8 & 8x - 8x = 0
\end{align*}
\]

\(8=8\) is our end result and it is a true statement. Because it is a true statement, we know that these 2 equations represent the same line. Therefore, this system of equations has an infinite number of solutions because every point on the line is a solution to the system.
6. Describe the graph for the following system of equations:

\[ 4y = 8x + 8 \]
\[ -2x + y = 7 \]

In order to describe the graph, we will need to solve the system of equations. Both equations would be easy to rewrite in \( y = \ldots \) form. You could divide all terms by 4 in the first equation: 

\[ \frac{4y}{4} = \frac{8x + 8}{4} \]
\[ y = 2x + 2 \]

OR you could add 2x to both sides to rewrite the second equation: 

\[ -2x + 2 + y = 7 + 2x \]
\[ y = 2x + 7 \]

**You could stop here if you look closely at the slope of each equation. Notice that the slopes are the same and they have different y-intercepts! If the equations have the same slope and different y-intercepts then the lines are parallel.**

If you don’t recognize this and you go on to solve using the substitution method, here’s what happens: Now pick one and substitute it into the other equation. I am going to substitute \( y = 2x + 2 \) into the second equation.

\[ -2x + 2 = 7 \]
\[ -2x + 2x + 2 = 7 \] Substitute \( 2x + 2 \) for \( y \)
\[ 2 = 7 \]
\[ -2x + 2x = 0 \]

I am left with \( 2 = 7 \) which is not a true statement. Therefore, this system does not have a solution. In this case the graph would show two parallel lines.
7. Find the solution to the system of equations and describe the graph.
\[
\begin{align*}
x &= 2y + 1 \\
3x + y &= 10
\end{align*}
\]

The first equation is already set up to substitute into the second equation. It is written as: 
\[x = \ldots\]

\[
\begin{align*}
3x + y &= 10 \\
3(2y + 1) + y &= 10 \quad \text{Substitute } 2y + 1 \text{ for } x. \\
6y + 3 + y &= 10 \quad \text{Substitute } 1 \text{ for } y \text{ into the 1st equation:} \\
7y + 3 &= 10 \quad \text{Distribute the } 3. \\
7y + 3 - 3 &= 10 - 3 \quad \text{Combine like terms: } 6y + y = 7y \\
7y &= 7 \quad \text{Subtract } 3 \text{ from both sides} \\
7y &= 7 \quad \text{Divide by } 7 \text{ on both sides} \\
y &= 1 \quad \text{The } y \text{ coordinate is } 1 \quad x = 3 \quad \text{The } x \text{ coordinate is } 3
\end{align*}
\]

The solution to this system of equations is \((3,1)\). The graph would show 2 lines intersecting at the point \((3,1)\).

8. Which ordered pair is the solution to the system of equations?
\[
\begin{align*}
y &= 4x + 2 \\
-8x + 2y &= 8
\end{align*}
\]

A. \((2,8)\)  
B. \((4,8)\)  
C. \((0,0)\)  
D. None of the Above

Since the first equation is already written as \(y = \ldots\), I am going to substitute \(4x + 2\) into the second equation.

\[
\begin{align*}
-8x + 2y &= 8 \\
-8x + 2(4x + 2) &= 8 \quad \text{Substitute } 4x + 2 \text{ for } y. \\
-8x + 8x + 4 &= 8 \quad \text{Distribute the } 2. \\
4 &= 8 \quad -8x + 8x = 0
\end{align*}
\]

\(4=8\) is not a true statement, therefore there are no solutions to this system of equations. The answer is none of the above. The answer should read: There are no solutions.
9. Find the solution to the system using substitution. Then check your answer by graphing.

\[ y = 2x - 8 \]
\[ x + y = 7 \]

Since the first equation is already set up as \( y = \ldots \), I am going to substitute \( 2x - 8 \) into the second equation for \( y \).

\[ x + y = 7 \]
\[ x + (2x - 8) = 7 \]
\[ 3x - 8 = 7 \]

Combine like terms: \( x + 2x = 3x \)

Add 8 to both sides.

\[ 3x - 8 + 8 = 7 + 8 \]
\[ 3x = \frac{15}{3} \]
\[ x = 5 \]

The \( x \) coordinate is 5

Substitute 5 for \( x \) into the first equation:

\[ Y = 2(5) - 8 \]
\[ Y = 2 \]

The \( y \) coordinate is 2

The solution to this system of equations is: \( (5, 2) \)

The first equation is already written in slope intercept form: \( y = 2x - 8 \). This is easy to graph.

The second equation is in standard form and must be rewritten in slope intercept form or you must find the \( x \) and \( y \) intercepts.

For this problem, it would be very easy to do either. I'll show you both ways!

**#1 Slope Intercept Form:**

\[ x + y = 7 \]
\[ x - x + y = 7 - x \]
\[ y = -x + 7 \]

Slope intercept form

**#2 X and Y intercept:**

\[ X \text{ intercept} \quad \text{Let } y = 0 \]
\[ x + 0 = 7 \]
\[ x = 7 \]

\[ Y \text{ intercept} \quad \text{Let } x = 0 \]
\[ 0 + y = 7 \]
\[ y = 7 \]
10. Which answer best describes the solution to the following system of equations?

\[ y = 4x - 2 \]
\[ y = 3x + 3 \]

A. (5, 18)
B. (18, 5)
C. (-2, 3)
D. None of the above

\[ \begin{align*}
y &= 4x - 2 \\
y &= 3x + 3 \\
4x - 2 &= 3x + 3 & \text{Substitute } 4x - 2 \text{ into the second equation for } y. \\
4x - 3x - 2 &= 3x - 3x + 3 & \text{Subtract } 3x \text{ from both sides.} \\
x - 2 &= 3 & \text{Simplify } (4x - 3x = x) \\
x - 2 + 2 &= 3 + 2 & \text{Add } 2 \text{ to both sides.} \\
x &= 5 & \\
\end{align*} \]

The answer to this system of equations is (5, 18)

11. Solve the following system of equations using substitution.

\[ y = \frac{3}{4}x - 6 \]
\[ y = \frac{1}{2}x - 4 \]

\[ \begin{align*}
y &= \frac{3}{4}x - 6 \\
y &= \frac{1}{2}x - 4 \\
3/4x - 6 &= 1/2x - 4 & \text{Substitute } 3/4x - 6 \text{ for } y. \\
4[3/4x - 6] &= [1/2x - 4]4 & \text{Multiply all terms by } 4 \text{ to get rid of fractions.} \\
3x - 24 &= 2x - 16 & \text{Subtract } 2x \text{ from both sides} \\
3x - 2x - 24 &= 2x - 2x - 16 & \\
x - 24 &= -16 & \text{Simplify } (3x - 2x = x) \\
x - 24 + 24 &= -16 + 24 & \text{Add } 24 \text{ to both sides} \\
x &= 8 & \text{The solution to this system of equations is } (8, 0) \\
\end{align*} \]
12. You are buying pizzas for a luncheon. Pizza Palace charges $5.50 a pizza and a $10 delivery and set up fee. Jerry’s Pizza charges $6.00 a pizza and a $7 delivery and set up fee.

• Write a linear system of equations to represent this situation.

• How many pizzas would you need to buy in order for the two companies to charge the same amount? How much would the pizzas cost?

• If you needed to order 10 pizzas, which company would be the best option in terms of cost? Explain your answer.

In this problem, I am given information about two different Pizza Companies. For both companies I am given a rate ($ per pizza) and a constant (delivery fee). Therefore, I can write an equation for both companies in slope intercept form.

Let \( x \) = number of pizzas \hspace{1cm} \text{Let} \ y = \text{total cost}

**Pizza Palace:** \( y = 5.50x + 10 \)

**Jerry’s Pizza:** \( y = 6x + 7 \)

In order to find out how many pizzas I would need to buy in order for the two companies to charge the same amount, I will need to solve the system of equations.

\[
\begin{align*}
Y = 5.50x + 10 & \\
Y = 6x + 7
\end{align*}
\]

\[
\begin{align*}
5.50x + 10 &= 6x + 7 & \text{Substitute 5.5x + 10 for y.} \\
5.50x - 5.50x + 10 &= 6x - 5.50x + 7 & \text{Subtract 5.50 x from both sides.} \\
10 &= .50x + 7 & \text{Simplify (6x – 5.5x = .5x)} \\
10 - 7 &= .50x + 7 - 7 & \text{Subtract 7 from both sides.} \\
3 &= .50x & \text{Simplify (10 -7 = 3)} \\
3 &= .50x & \text{Divide both sides by .50} \\
\frac{3}{.50} &= x \\
6 &= x \\
\end{align*}
\]

**Step 2:** Substitute 6 for \( x \).

\[
\begin{align*}
x &= 6 & \text{y} &= 6(6) + 7 \\
y &= 43
\end{align*}
\]

The solution is \((6, 43)\).

You would need to order 6 pizzas for the companies to charge the same amount, $43. If you needed to order 10 pizzas, it would be cheaper to order from Pizza Palace because they would only charge $65 and Jerry’s charges $67.

**Pizza Palace:** \( y = 5.5(10) + 10 \)

\( Y = 65 \)

**Jerry’s:** \( y = 6(10) + 7 \)

\( y = 67 \)
1. Solve the system of equations using the substitution method. Justify your answer mathematically. (3 points)

\[ \begin{align*}
Y &= \frac{1}{3}x + 2 \\
Y &= 3x - 5
\end{align*} \]

Since both equations are written in slope intercept form, it doesn’t matter which one your substitute.

\[
\begin{align*}
3x - 5 &= \frac{1}{3}x + 2 \\
3(3x - 5) &= 3\left(\frac{1}{3}x + 2\right) \\
9x - 15 &= x + 6 \\
9x - x - 15 &= x - x + 6 \\
8x - 15 &= 6 \\
8x - 15 + 15 &= 6 + 15 \\
8x &= 21 \\
\frac{8x}{8} &= \frac{21}{8} \\
x &= \frac{21}{8} \text{ or } x = 2.625
\end{align*}
\]

This is the value of the x coordinate.

Now we must substitute 2.625 into one of the equations in order to solve for y.

\[ Y = 3x - 5 \]

\[ Y = 3(2.625) - 5 \]

This is the value of the y-coordinate.

The solution to this system of equations is (2.625, 2.875)

**JUSTIFICATION:** The solution must prove to be correct in BOTH equations.

\[
\begin{align*}
\#1: & \quad y = 3x - 5 \quad 2.875 = 3(2.625) - 5 \\
& \quad 2.875 = 2.875 \\
\#2: & \quad y = \frac{1}{3}x + 2 \quad 2.875 = \frac{1}{3}(2.625) + 2 \\
& \quad 2.875 = 2.875
\end{align*}
\]

Since the solution is correct in both equations, our solution to the system is correct!
2. Solve the system of equations using the substitution method. If you were to graph this system of equations, describe what the graph would look like. (3 points)

\[ \begin{align*}
Y &= -\frac{2}{3}x + 6 \\
2x + 3y &= -12
\end{align*} \]

Since one equation is already in the form of \( y = \), we will substitute this expression into equation #2

\[ \begin{align*}
2x + 3\left(-\frac{2}{3}x + 6\right) &= -12 \\
2x + -2x + 18 &= -12 \tag{Distribute the 3 throughout the parenthesis.}
\end{align*} \]

\[ 18 = -12 \tag{Simplify: \( 2x + -2x = 0 \); therefore we have no variable.} \]

Since the expression \( 18 = -12 \) is not true, we know that this system of equations does not have a solution. Systems of equations that do not have a solution are parallel lines when graphed. Parallel lines never intersect and that is why there is no solution to the system of equations.

The graph for this system would show two parallel lines.

3. Hannah has $310 and saves $8 per week. Kara has $220 and saves $12 per week. After how many weeks will Hannah and Kara have the same amount of money? (4 points)

- Write a system of equations to describe this situation.

Let \( x \) = the number of weeks and \( y \) = the total amount of money saved.

Hannah: \( y = 8x + 310 \) \tag{8 is the slope since it contains the key word “per”. She has 310, so this is a constant.}

Kara: \( y = 12x + 220 \) \tag{12 is the slope since it contains the key word “per”. She has 220, so this is a constant.}
• Solve the system of equations using the substitution method.

Substitute the 2nd equation into the 1st equation.

\[ y = 8x + 310 \]
\[ y = 12x + 220 \]
\[ 12x + 220 = 8x + 310 \]

Now we have an equation with variables on both sides. Solve the equation for \( x \).

\[ 12x - 8x + 220 = 8x - 8x + 310 \] Subtract 8x from both sides
\[ 4x + 220 = 310 \] Simplify: \( 12x - 8x = 4x \)
\[ 4x + 220 - 220 = 310 - 220 \] Subtract 220 from both sides
\[ 4x = 90 \] Simplify: \( 310 - 220 = 90 \)
\[ 4x/4 = 90/4 \] Divide by 4: \( 8 - 8x + 310 \)
\[ 4x + 220 = 310 \] Simplify: \( 12x - 8x = 4x \)
\[ 4x + 220 - 220 = 310 - 220 \] Subtract 220 from both sides
\[ 4x = 90 \] Simplify: \( 310 - 220 = 90 \)
\[ 4x/4 = 90/4 \] Divide by 4 on both sides
\[ x = 22.5 \]

**After 22.5 weeks, Hannah and Kara will have the same amount of money.**

• How much money will Hannah and Kara have when their savings are the same?

In order to find out how much money they will have when their savings are the same, we will substitute 22.5 for \( x \) into both equations. This will also allow us to determine whether our answer is correct. They should both have the same amount when substituting 22.5.

Hannah: \( y = 8x + 310 \)

\[ Y = 8(22.5) + 310 \]
\[ Y = 490 \]

Kara: \( y = 12x + 220 \)

\[ y = 12(22.5) + 220 \]
\[ y = 490 \]

**They will both have $490 dollars in their savings account after 22.5 weeks.**
Systems of Equations – Mid Chapter Quiz

1. Estimate the solution to the system of equations.
   
   A. (-4,2)
   B. (2, -4)
   C. (4, -2)
   D. (-2, 4)

2. John graphed the system of equations below.

   \[ y = 8x - 9 \]
   \[ y = 8x + 2 \]

   Which of these statements best describes the relationship between the two lines?

   A. They have no points in common.
   B. They have one point in common.
   C. They have two points in common.
   D. They have an infinite number of points in common.
3. Jerry decided to solve the following system of equations using the substitution method.

\[
\begin{align*}
5x - 2y &= 10 \\
3x + y &= 12
\end{align*}
\]

Jerry isolates \( y \) in the second equation. What expression will he substitute for \( y \) into the first equation?

A. \( 3x + 12 \)  
B. \( 9x \)  
C. \( 12 - 3x \)  
D. \( 12 + 3x \)

4. Which display represents the following system?

\[
\begin{align*}
-x + 5y &= 5 \\
-2x + 10y &= 10
\end{align*}
\]

A          B       C    D

5. What is the \( y \)-coordinate for the following system of equations?

\[
\begin{align*}
x - 2y &= 7 \\
x + 3y &= -3
\end{align*}
\]

A. -2  
B. 0  
C. 2  
D. 1
6. Find the slope of a line that is parallel to: $7x - 3y = 8$

7. Find the sum of the $x$ and $y$ coordinates of the solution to the following system of equations.

\[
\begin{align*}
    x - y &= 2 \\
    2x + y &= 1
\end{align*}
\]
**Systems of Equations – Mid Chapter Quiz – Answer Key**

1. Estimate the solution to the system of equations. (1 point)

   A. (-4, 2)
   B. (2, -4)
   C. (4, -2)
   D. (-2, 4)

   The point of intersection is the solution to the system of equations. Therefore the solution is (-4, 2)

2. John graphed the system of equations below. (1 point)

   \[ y = 8x - 9 \]
   \[ y = 8x + 2 \]

   Which of these statements best describes the relationship between the two lines?

   A. They have no points in common.
   B. They have one point in common.
   C. They have two points in common.
   D. They have an infinite number of points in common.

   The two equations have the same slope (8) and different y-intercepts. Whenever two equations have the same slope, and different y-intercepts they will be parallel lines. Parallel lines will never intersect, therefore, there is no solution to the system of equations. If the lines never intersect, they will not have any points in common.
3. Jerry decided to solve the following system of equations using the substitution method. (1 point)

\[\begin{align*}
5x - 2y &= 10 \\
3x + y &= 12
\end{align*}\]

Jerry isolates \( y \) in the second equation. What expression will he substitute for \( y \) into the first equation?

A. \( 3x + 12 \)  
B. \( 9x \)  
C. \( 12 - 3x \)  
D. \( 12 + 3x \)

If Jerry isolates \( y \) in the second equation, then he solves that equation for \( y \).

\[\begin{align*}
3x + y &= 12 \\
3x - 3x + y &= 12 - 3x \\
y &= 12 - 3x
\end{align*}\]

This is the expression he will substitute into the 1st equation.

4. Which display represents the following system? (1 point)

\[\begin{align*}
-x + 5y &= 5 \\
-2x + 10y &= 10
\end{align*}\]

We need to decide which graph represents this system of equations. Since both equations are written in standard form, we could either find the intercepts of both equations or we could rewrite them in slope intercept form. Since the graphs are small, it may be difficult to estimate where the lines intercept the \( x \) and \( y \) axis, so I am going to rewrite the equations in slope intercept form. (This form may also relay other important information 😊)

\[\begin{align*}
-x + 5y &= 5 \\
-2x + 10y &= 10
\end{align*}\]

\[\begin{align*}
-x + x + 5y &= 5 + x \\
-2x + 2x + 10y &= 10 + 2x
\end{align*}\]

\[\begin{align*}
5y &= x + 5 \\
5y &= x + 5
\end{align*}\]

\[\begin{align*}
5 &= 5 & 5 \\
10y &= 2x + 10 & 10 & 10
\end{align*}\]

\[\begin{align*}
y &= 1/5x + 1 \\
y &= 1/5x + 1
\end{align*}\]

The equations are exactly the same when written in slope intercept form. This means the lines lay on top of each other. Therefore, the answer must be either A or B.

Since the slope is positive \((1/5)\), the correct answer must be A. Letter B has a negative slope.
5. What is the y-coordinate for the following system of equations?  (1 point)

\[
\begin{align*}
x - 2y &= 7 \\
x + 3y &= -3
\end{align*}
\]

- \(A\) -2
- \(B\) 0
- \(C\) 2
- \(D\) 1

In order to find the y-coordinate, we must solve the system of equations. I will use substitution in order to solve this system.

1. I will solve the first equation for \(x\).
   \[
x - 2y + 2y = 7 + 2y \\
x = 7 + 2y
\]

2. Now we will substitute \(x = 7 + 2y\) into the second equation.
   \[
x + 3y = -3 \\
7 + 2y + 3y = -3 \\
7 + 5y = -3 \\
7 - 7 + 5y = -3 - 7 \\
5y = -10 \\
5y/5 = -10/5 \\
y = -2
\]

The y-coordinate for this system of equations is -2.

6. Find the slope of a line that is parallel to: \(7x - 3y = 8\)  (2 points)

There are two things that we need to consider in this problem:

1. We need to know the characteristics of parallel lines.
   
   We know that parallel lines have the same slope, and different y-intercepts. So, a line that is parallel will have the same slope as this line.

2. We need to be able to identify the slope in the equation.
   
   In order to identify the slope, the equation must be written in slope intercept form. Therefore, we need to rewrite this equation in slope intercept form.

\[
7x - 3y = 8
\]

\[
7x - 7x - 3y = 8 - 7x \\
-3y = -7x + 8 \\
-3 -3 -3
\]

\[
Y = 7/3x - 8/3
\]

The slope of this line is \(7/3\). Therefore, a line parallel would also have a slope of \(7/3\).

The slope of a line parallel to \(7x - 3y = 8\) is \(7/3\).
7. Find the sum of the x and y coordinates of the solution to the following system of equations. (3 points)

\[
\begin{align*}
\begin{align*}
x - y &= 2 \\
2x + y &= 1
\end{align*}
\end{align*}
\]

This is one of those tricky questions that test your knowledge of math vocabulary. The question asks for the sum of the x and y coordinates of the solution. Therefore, we need to find the solution (which is the x and y coordinates) and then add them together to find the sum.

1. We need to solve this system. I will use substitution and solve the first equation for x.

\[
\begin{align*}
x - y &= 2 \\
x &= y + 2
\end{align*}
\]

2. Substitute \(x = y + 2\) into the second equation.

\[
\begin{align*}
2x + y &= 1 \\
2(y + 2) + y &= 1 \\
2y + 4 + y &= 1 \\
3y + 4 &= 1 \\
3y + 4 - 4 &= 1 - 4 \\
3y &= -3 \\
\frac{3y}{3} &= \frac{-3}{3} \\
y &= -1 \text{ - The y coordinate is -1}
\end{align*}
\]

3. Substitute -1 for y into the equation: \(x = y + 2\) (This is the easiest method. You could substitute into any equation given.)

\[
\begin{align*}
x &= y + 2 \\
x &= -1 + 2 \\
x &= 1 \text{ - The x coordinate is 1}
\end{align*}
\]

Double check your answers by substituting:

\[
\begin{align*}
x - y &= 2 & 2x + y &= 1 \\
1 - (-1) &= 2 & 2(1) + (-1) &= 1 \\
1 + 1 &= 2 & 2 + (-1) &= 1 \\
2 &= 2 & 1 &= 1
\end{align*}
\]

Since the x coordinate is 1 and the Y coordinate is -1, we need to add them together in order to find the sum.

\(-1 + 1 = 0\)

The sum of the x and y coordinates is 0. This is the final answer!
### Lesson 3: Using Linear Combinations to Solve a System of Equations

**Steps for Using Linear Combinations to Solve a System of Equations**

1. __________________________________________________________________________

2. __________________________________________________________________________

3. __________________________________________________________________________

4. __________________________________________________________________________

5. __________________________________________________________________________

### Example 1

Solve the following system using the linear combinations method.

\[ 2x - 3y = 12 \quad \& \quad 4x + 3y = 6 \]
Example 2

Solve the following system using the linear combinations method.

\[ y = 4x - 2 \quad \text{&} \quad 8x - 2y = -12 \]

Example 3

Solve the following system using the linear combinations method.

\[ 2x - 3y = 3 \quad \text{&} \quad 6x - 9y = 9 \]
Lesson 3: Using Linear Combinations Method

1. Which of the following ordered pairs is a solution to the system of equations?
   \[ 4x + 3y = 5 \]
   \[ 2x - 3y = 7 \]
   
   A. (5,7)  
   B. (2, -1)  
   C. (-1,2)  
   D. None of the Above.

2. Use Linear Combinations to find the solution to the following system of equations.
   \[ 3x + 2y = 8 \]
   \[ x - 2y = 4 \]

3. Which statement best describes the solution to the following system of equations?
   \[ 5x - y = -2 \]
   \[ 10x - 2y = -4 \]
   
   A. (-2,-4) is the solution.  
   B. There is no solution, the lines are parallel.  
   C. There are an infinite number of solutions, they are the same line.  
   D. None of the Above.

4. Solve the following system of equations using linear combinations.
   \[ x - 2y = -10 \]
   \[ 3x - y = 0 \]
5. Which ordered pair is the solution to the following system of equations?

\[ 3x + 2y = 10 \]
\[ 2x + 5y = 3 \]

A. (10,3)  
B. (-1,4)  
C. (4,-1)  
D. None of the Above

6. Describe the graph of the solution for the following system of equations.

\[ x - 2y = 10 \]
\[ 2x - 4y = 20 \]

7. Find the solution to the following system of equations. Explain what the solution means.

\[ 3x + 2y = 8 \]
\[ 4x - 3y = 5 \]

8. Find the solution to the following system of equations.

\[ 4x - 2y = 2 \]
\[ 3x - 3y = 9 \]
Directions: Solve each system of equations using the Linear Combinations method. Each problem is worth 3 points.

1. \[ 2a + 8b = 2 \]
   \[ -2a - 4b = 6 \]

2. \[ x + 3y = 18 \]
   \[ 2x + y = 11 \]

3. \[ 2x - 4y = -4 \]
   \[ 3x + 3y = 3 \]

4. \[ 5x - y = -2 \]
   \[ 10x - 2y = 8 \]
Lesson 3: Using Linear Combinations Method – Answer Key

1. Which of the following ordered pairs is a solution to the system of equations?
   \[ 4x + 3y = 5 \]
   \[ 2x - 3y = 7 \]

   A. (5,7)  
   B. (2, -1)  
   C. (-1,2)  
   D. None of the Above.

   \[ 4x + 3y = 6 \]  We already have opposite terms so we can add!  
   \[ 2x - 3y = 7 \]
   \[ 6x = 12 \]  
   \[ x = 2 \]

   \[ 6x = 12 \]  
   \[ 6 \]

   Step 2: Substitute 2 for x into either equation.

   \[ 2x - 3y = 7 \]
   \[ 2(2) - 3y = 7 \]
   \[ 4 - 3y = 7 \]

   \[ 4 - 4 = 7 - 4 \]  Subtract 4 from both sides
   \[ -3y = 3 \]
   \[ -3 \]

   \[ y = -1 \]  (2,-1) is the solution

2. Use Linear Combinations to find the solution to the following system of equations.
   \[ 3x + 2y = 8 \]
   \[ x - 2y = 4 \]

   \[ 3x + 2y = 8 \] We already have opposite terms, so we can add.
   \[ x - 2y = 4 \]

   \[ 3x = 12 \]  
   \[ 4 \]

   \[ x = 3 \]

   Step 2: Substitute 3 for x into either equation.

   \[ x - 2y = 4 \]
   \[ 3 - 2y = 4 \]  Substitute 3 for x.

   \[ 3 - 3 - 2y = 4 - 3 \]  Subtract 3 from both sides
   \[ -2y = 1 \]
   \[ -2 \]

   \[ y = -1/2 \]  The solution to the system is (3, -1/2)
3. Which statement best describes the solution to the following system of equations?

\[5x - y = -2\]
\[10x - 2y = -4\]

-2[5x – y = -2] \[\rightarrow\] -10x + 2y = 4
10x – 2y = -4 \[\rightarrow\] 10x – 2y = -4
0 + 0 = 0

We don’t have opposite terms, so I multiplied the first equation by -2, to make the y term 2y. 0 = 0 does make sense.

After adding the two equations, I am left with 0 = 0. This statement does make sense. Therefore, there is an infinite number of solutions to this system. The lines are equivalent.

A. (-2, -4) is the solution.
B. There is no solution, the lines are parallel
C. There are an infinite number of solutions, they are the same line.
D. None of the Above.

4. Solve the following system of equations using linear combinations.

\[x – 2y = -10\]
\[3x – y = 0\]

-3[x – 2y = -10] \[\rightarrow\] 3x + 6y = 30
3x – y = 0 \[\rightarrow\] 3x – y = 0
5y = 30
5
5

We did not have terms that were opposites, so I multiplied the first equation by -3 to make the x term, -3x. Add the equation together and you get 5y = 30.

\[5y = 30\]
\[5\]
\[5\]

Solve for y. Divide by 5 on both sides.

\[y = 6\]

**Step 2: Substitute 6 for y into either equation.**

\[3x – y = 0\]
I chose equation #2 because y didn’t have a coefficient. It doesn’t matter which equation you choose. You’ll end up with the same answer!

\[3x – 6 = 0\]
Add 6 to both sides.
\[3x = 6\]
Simplify: 0 + 6 = 6
\[3\]
\[3\]
\[x = 2\]

The solution to this system of equations is: (2, 6)
5. Which ordered pair is the solution to the following system of equations?

\[3x + 2y = 10\]
\[2x + 5y = 3\]

A. (10,3)
B. (-1,4)
C. (4,-1)
D. None of the Above

\[-2(3x + 2y = 10) \rightarrow -6x - 4y = -20\]
\[3(2x + 5y = 3) \rightarrow 6x + 15y = 9\]
\[11y = -11\]
\[y = -1\]

**Step 2: Substitute -1 for y into either equation.**

\[2x + 5(-1) = 3\]
\[2x - 5 = 3\]
\[2x = 8\]
\[x = 4\]

The solution to this system of equations is (4, -1)

6. Describe the graph of the solution for the following system of equations.

\[x - 2y = 10\]
\[2x - 4y = 20\]

\[-2[x - 2y = 10] \rightarrow -2x + 4y = -20\]
\[2x - 4y = 20 \rightarrow 2x - 4y = 20\]
\[0 = 0\]

I multiplied the first equation by -2 to make opposite terms (-2x & 2x)

Add the equations together.

We can’t go any further and 0 = 0 is a true statement. Therefore, these two equations are equivalent. They are the same line lying on top of each other. There are an infinite number of solutions to the system of equations.
7. Find the solution to the following system of equations. Explain what the solution means.

\[
\begin{align*}
3x + 2y &= 8 \\
4x - 3y &= 5
\end{align*}
\]

\[
\begin{align*}
3[3x + 2y = 8] &\rightarrow 9x + 6y = 24 \\
2[4x - 3y = 5] &\rightarrow 8x - 6y = 10
\end{align*}
\]

I multiplied the first equation by 3 and the second equation by 2 to make the y terms opposites (6y \& -6y)

\[
\begin{align*}
17x &= 34 \\
\frac{17}{17} &= \frac{34}{17} \\
x &= 2
\end{align*}
\]

Step 2: Substitute 2 for x into either equation.

\[
\begin{align*}
4x - 3y &= 5 \\
4(2) - 3y &= 5 \\
8 - 3y &= 5 \\
-8 - 3y &= 5 -8 \\
-3y &= -3 \\
\frac{-3y}{-3} &= \frac{-3}{-3} \\
y &= 1
\end{align*}
\]

The solution (2,1) is the point where the two lines will intersect on a graph. It means that (2,1) is a solution for both equations. This is the only solution that both equations have in common.

---

8. Find the solution to the following system of equations.

\[
\begin{align*}
4x - 2y &= 2 \\
3x - 3y &= 9
\end{align*}
\]

\[
\begin{align*}
-3[4x - 2y = 2] &\rightarrow -12x + 6y = -6 \\
2[3x - 3y = 9] &\rightarrow 6x - 6y = 18
\end{align*}
\]

I multiplied the first equation by -3 and the second equation by 2 in order to make the y terms opposites (6y \& -6y).

\[
\begin{align*}
-6x &= 12 \\
\frac{-6}{-6} &= \frac{12}{-6} \\
x &= -2
\end{align*}
\]

Step 2: Substitute -2 for x into either equation and solve for y.

\[
\begin{align*}
4x - 2y &= 2 \\
4(-2) - 2y &= 2 \\
-8 - 2y &= 2 \\
-8 + 8 - 2y &= 2 + 8 \\
-2y &= 10 \\
\frac{-2y}{-2} &= \frac{10}{-2} \\
y &= -5
\end{align*}
\]

Solution is (-2,-5)
Directions: Solve each system of equations using the Linear Combinations method. Each problem is worth 3 points.

1. \(2a + 8b = 2\)
   \(-2a - 4b = 6\)

The first terms are already opposites so we can go ahead and add:

\[
\begin{align*}
4b &= 8 & \text{Add the 2 equations. Now solve for } b. \\
4b/4 &= 8/4 & \text{Divide by 4 on both sides} \\
b &= 2 & b = 2, now we must find } a \text{ by substituting.}
\end{align*}
\]

\[
\begin{align*}
2a + 8b &= 2 & \text{Choose one of the equation to substitute back into.} \\
2a + 8(2) &= 2 & \text{Substitute 2 for } b. \\
2a + 16 &= 2 & \text{Simplify: } 8(2) = 16 \\
2a - 16 &= -14 & \text{Subtract 16 from both sides} \\
2a/2 &= -7 & \text{Divide by 2 on both sides} \\
a &= -7 & b = 2 \text{ and } a = -7
\end{align*}
\]

2. \(x + 3y = 18\)
   \(2x + y = 11\)

These two equations do not have opposite terms, so we must multiply the first equation by -2 to create opposite x terms: (-2x & 2x). (Or you could multiply the second equation by -3 to create opposite y terms: (3y &-3y)

\[
\begin{align*}
-2(x + 3y) &= (18) \times -2 & \text{Multiply all terms by -2} \\
2x + y &= 11 & \text{Keep the same} \\
2x - 6y &= -36 & \text{Multiply all terms by -2} \\
2x + y &= 11 & \text{Add the equations} \\
-5y &= -25 & \text{Divide by -5 on both sides} \\
-5y/-5 &= -25 / -5 & y = 5, now solve for } x. \\
y &= 5 & \text{Use this equation to substitute 5 for } y \\
2x + y &= 11 & \text{Substitute 5 for } y \\
2x + 5 &= 11 & \text{Subtract 5 from both sides} \\
2x &= 6 & \text{Simplify: } 11-5 = 6 \\
2x/2 &= 6/2 & \text{Divide by 2 on both sides} \\
x &= 3 & x = 3 \text{ and } y = 5
\end{align*}
\]
3. \[2x - 4y = -4\]
\[3x + 3y = 3\]

We need to create opposites and the only way to do this is to multiply the first equation by \(-3\) and the second equation by \(2\). This will create opposite x terms: \((-6x, 6x)\). OR you could create opposite y terms by multiplying the first equation by \(3\) and the second equation by \(4\).

\[-3(2x - 4y) = -3(-4) \quad \rightarrow \quad -6x + 12y = 12 \quad \text{Multiply by } -3\]
\[2(3x+3y) = 2(3) \quad \rightarrow \quad 6x + 6y = 6 \quad \text{Multiply by } 2\]

\[18y = 18 \quad \text{Now we can solve for } y.\]
\[18y/18 = 18/18 \quad \text{Divide by } 18 \text{ on both sides.}\]
\[y = 1 \quad \text{y = 1, now we need to solve for } x.\]

\[3x + 3y = 3 \quad \text{We can use this equation to solve for } x.\]
\[3x + 3(1) = 3 \quad \text{Substitute } 1 \text{ for } y.\]
\[3x + 3 = 3 \quad \text{Simplify: } 3(1) = 3\]
\[3x + 3-3= 3-3 \quad \text{Subtract 3 from both sides}\]
\[3x = 0 \quad \text{Simplify: } 3-3 = 0\]
\[3x/3 = 0/3 \quad \text{Divide by } 3 \text{ on both sides}\]
\[x = 0 \quad x = 0 \text{ and } y = 1\]

4. \[5x - y = -2\]
\[10x - 2y = 8\]

We do not have opposite terms, so we must multiply the first equation by \(-2\) in order to create opposite terms.

\[-2(5x - y) = -2(-2) \quad \rightarrow \quad -10x + 2y = 4 \quad \text{Multiply by } -2\]
\[10x - 2y = 8 \quad \rightarrow \quad 10x - 2y = 8 \quad \text{Keep the same}\]
\[0 = 12 \quad \text{Add the equations}\]

This statement does not make sense, and we do not have a variable to solve for. Since \(0\) is not equal to \(12\), we know that there are no solutions to this system of equations. These two lines are parallel and there are no solutions.
**Bonus Lesson – Which Method Should I Use?**

Well, now that you’ve learned three different ways for solving a system of equations, how do you know which method is best?

Let’s take a look at each of the three different ways for solving a system of equations:

### Method 1: Graphing

Graphing is a great method for solving a system if you have graph paper!

If the equations are written in slope intercept form (and the numbers are small enough to fit on your graph), you can graph them pretty quickly to find the solution.

If the equations are written in standard form, you can graph them using the x and y intercept method or you can rewrite them in slope intercept form. Either way, you will arrive at the solution!

If you don’t have graph paper handy – this is not the method to use!

### Method 2: Using the Substitution Method

The substitution method can be used to solve any system of equations but it’s the easiest method to use when...

- One or more of the equations are written in \( y = mx + b \) form.

  If an equation is already written in this form, you can substitute into the 2\(^{nd} \) equation and solve.

- One of the equations is written as \( x = ____ \). In this case, you can go ahead and substitute this equation into the 2\(^{nd} \) equation and solve.

- At least one of your equations has a coefficient of 1:

  - \( 3x + y = 2 \)

    Y has a coefficient of 1. You can subtract 3x from both sides and the equation will be written as \( y = -3x + 2 \). Then it is ready to substitute.

  - \( x + 2y = 3 \)

    X has a coefficient of 1. You can subtract 2y from both sides and the equation will be written as \( x = -2y + 3 \). Then it is ready to substitute.
Method 3: Using Linear Combinations (Addition Method)

Linear combinations can be used to solve any system of equations, but it’s the easiest method to use when...

- Both equations are written in standard form.

3x + 2y = 8
2x + 4y = 6

Some students favor a particular method and use it to solve most systems of equations. That is fine! It is nice to be able to use both methods and to know which method will save you time based on the problem that you are solving!

The tricky part is knowing what the solutions will look like when the lines are parallel or the same line!

Now you are ready to apply all of the knowledge to solve real world problems! You can use any method you like to solve these problems!
Lesson 4: System of Equations Word Problems

Example 1
John has $300 in his savings account and he saves $15 per month. Joe saves $50 per month, and he started with $125 in his savings account. In how many weeks with John and Joe have equal savings?
Example 2

A test was given to an Algebra class. The 40 question test is worth 90 points. There were short answer questions worth 2 points and extended response questions worth 3 points each.

- Write a system of equations that represents this situation.
- How many short answer questions are on the test?
- How many extended response questions are on the test?
## Word Problems Organizer

<table>
<thead>
<tr>
<th>What I Know</th>
<th>Define Your Variable(s.)</th>
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<th>Write a Verbal Model &amp; Substitute</th>
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Lesson 4: Systems of Equations – Real World Problems

1. A test was given to a Pre-Algebra class. The 30 question test was worth 50 points. There were multiple choice questions worth 1 point each and short answer questions worth 3 points each. Let x represent the number of 1 point questions and y represent the number of 3 point questions. Which system of equations represents this situation?

A. \[ \begin{align*} x + y &= 50 \\ x + 3y &= 30 \end{align*} \]  
B. \[ \begin{align*} 3x + y &= 30 \\ x + y &= 50 \end{align*} \]  
C. \[ \begin{align*} x + y &= 30 \\ x + 3y &= 50 \end{align*} \]  
D. \[ \begin{align*} x + y &= 30 \\ 3x + 3y &= 50 \end{align*} \]

2. You are going to a baseball game at Camden Yards. One parking garage charges a flat fee of $10 and $0.50 per hour for parking. A parking garage across the street from the ball park charges a flat fee of $5 and $1.50 per hour for parking. Let x represent the number of hours spent parking and y represent the total cost. Which system of equations represents this situation?

A. \[ \begin{align*} x + 0.50y &= 10 \\ x + 1.50y &= 5 \end{align*} \]  
B. \[ \begin{align*} y &= 0.5x + 10 \\ y &= 1.5x + 5 \end{align*} \]  
C. \[ \begin{align*} 0.50x + y &= 10 \\ 1.50x + y &= 5 \end{align*} \]  
D. \[ \begin{align*} y &= 10x + 0.50 \\ y &= 5x + 1.50 \end{align*} \]

3. You are considering getting a new cell phone with texting privileges. *Cell Phone America* charges $40 a month and $0.39 per text message. *Call on Me* charges $30 a month and $0.49 per text message. After how many text messages would both companies charge the same amount?

A. 100  
B. 105  
C. 110  
D. 98
4. Lisa has $22 in her bank account and she deposits $11.50 each week. Janine has $118 in her bank account and she deposits $6.70 each week. In how many weeks, will Lisa have more money in her bank account?

A. 20      C. 19
B. 21      D. 22

5. The Lakers scored a total of 80 points in a basketball game. The Lakers made a total of 37 two-point and three-point baskets.
   • Write a linear system of equations that represents this situation.
   • How many two-point shots did the Lakers make? Justify your answer.
   • How many three-point shots did the Lakers make? Justify your answer.

6. The perimeter of an ice skating rink is 180 feet. The length of the rink is 10 feet longer than the width.
   • Write a linear system of equations that represents this situation.
   • What is the length and width of the ice skating rink? Explain how you determined your answer.
7. John is thinking of buying one of two cars. Car A will cost $17655. Average cost per year for fuel, maintenance, and repairs is $1230. Car B will cost about $15900. Fuel, maintenance, and repairs will cost $1425 per year.

- Write a system of equations to represent this situation.
- You expect to only keep the car about 5 years. Which car should you buy? Justify your answer.

8. Go Baby Go Inc. manufactures four-wheeled strollers and six-wheeled strollers. The company has 1800 wheels in stock and needs to make a total of 350 strollers.

- Write a system of equations that represents this situation.
- How many of each product can the company make? Justify your answer.
9. The local high school is selling pizza kits to raise money for the end of the year field trip. The high school pays $30 for a contract with the pizza company and $5 for each kit. The selling price of each kit is $9.

- Write a system of equations to represent this situation.
- How many kits must the high school sell in order to break even? Justify your answer.

10. Jerry wants to print a bunch of old pictures. He has decided upon two companies than can print the pictures for him.

<table>
<thead>
<tr>
<th>Company</th>
<th>Membership Fee</th>
<th>Price per photo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photome.com</td>
<td>$10</td>
<td>$0.10</td>
</tr>
<tr>
<td>Smile.com</td>
<td>$15</td>
<td>$0.06</td>
</tr>
</tbody>
</table>

- Write an equation to represent the cost for Photome.com.
- Write an equation to represent the cost for Smile.com.
- Jerry plans to print over 200 pictures. Which company should he choose? Justify your answer.

1. Missy has $200 and saves $50 a week. Marcus has $1800 and spends $50 a week. How many weeks from now will they have the same amount of money? How much money will they each have? (3 points)
2. A resort hotel is offering a special for the holiday weekend:
   Special #1: two nights and 1 meal for two for $215
   Special #2: three nights and 2 meals for two for $345

What price is the hotel charging per night and per meal? (3 points)

3. Jesse’s results on a standardized test show that his math score was 70 points higher than his reading score. His total score for the two parts is 1340. Let $m = \text{math score}$ and $r = \text{reading score}$. (4 points)
   • Write a system of equations for this situation.
   • Find Jesse’s math score and reading score.
   • Justify your answers using mathematics.
Lesson 4: Systems of Equations – Real World Problems (Answer Key)

1. A test was given to a Pre-Algebra class. The 30 question test was worth 50 points. There were multiple choice questions worth 1 point each and short answer questions worth 3 points each. Let $x$ represent the number of 1 point questions and $y$ represent the number of 3 point questions. Which system of equations represents this situation?

Identify the two pieces of information that you know: how many questions and the number of points!

$x = \text{number of 1 point questions.} \quad y = \text{number of 3 point questions.}$

1. Write an equation based on the information about the number of points.

$$x + y = 30$$

# of 1 pt. + # of 3 pt = total # of questions.

2. Write an equation based on the value of each question (the number of points).

$$x + 3y = 50$$

Correctly written as: $x + 3y = 50$

$1 \text{pt} \cdot \# \text{ of ques.} + 3 \text{pts} \cdot \# \text{ of ques.} = \text{total number of points.}$

A. $x + y = 50 \quad x + 3y = 30$

B. $3x + y = 30 \quad x + y = 50$

C. $x + y = 30 \quad x + 3y = 50$

D. $x + y = 30 \quad 3x + 3y = 50$
2. You are going to a baseball game at Camden Yards. One parking garage charges a flat fee of $10 and $0.50 per hour for parking. A parking garage across the street from the ball park charges a flat fee of $5 and $1.50 per hour for parking. Let \( x \) represent the number of hours spent parking and \( y \) represent the total cost. Which system of equations represents this situation?

I know information about two different parking garages. Let's write an equation to represent each parking garage.

For each parking garage I am given a rate and a flat fee, so I can write my equations in slope intercept form.

Parking garage 1: \( y = 0.50x + 10 \)

Parking Garage 2: \( y = 1.50x + 5 \)

A. \( x + 0.5y = 10 \)  \( x + 1.5y = 5 \)
B. \( y = 0.5x + 10 \)  \( y = 1.5x + 5 \)
C. \( 0.5x + y = 10 \)  \( 1.50x + y = 5 \)
D. \( y = 10x + 0.50 \)  \( y = 5x + 1.50 \)
3. You are considering getting a new cell phone with texting privileges. *Cell Phone America* charges $40 a month and $0.39 per text message. *Call on Me* charges $30 a month and $0.49 per text message. After how many text messages would both companies charge the same amount?

We know information about 2 different cell phone companies. Let's write two equations: 1 for each company.

We also see that for each company we are given a rate and a flat fee per month. Therefore, we can write our equations in slope intercept form.

Let \( x \) = the number of text messages
Let \( y \) = the total amount of the bill.

**Cell Phone America**: \( y = 0.39x + 40 \)

**Call on Me**: \( y = 0.49x + 30 \)

Now that we have two equations, we can solve the system of equations. The solution represents the number of text messages that need to be made in order for the two companies to charge the same amount. Therefore, this would answer our question.

Since both equations are written in slope intercept form, I am going to use the substitution method to solve the system of equations.

\[
\begin{align*}
Y &= 0.39x + 40 \\
y &= 0.49x + 30
\end{align*}
\]

\( 0.49x + 30 = 0.39x + 40 \)
Substitute \( 0.49x + 30 \) for \( y \) into the first equation. Now you have an equation with variables on both sides.

\( 0.49x - 0.39x + 30 = 0.39x - 0.39x + 40 \)
Subtract \( 0.39x \) from both sides.

\( 0.10x + 30 = 40 \)
Simplify \((0.49 - 0.39 = 0.1)\)

\( 0.10x + 30 - 30 = 40 - 30 \)
Subtract 30 from both sides.

\( 0.10x = 10 \)
Simplify \((40 - 30 = 10)\)

\( \frac{0.10x}{0.10} = \frac{10}{0.10} = 100 \)
Divide both sides by \( 0.10 \)

\( x = 100 \)
Simplify \((10/0.10 = 100)\)

\( x \) = the number of text messages, so for 100 text messages, the two companies would charge the same amount.

**Answer**: 100
4. Lisa has $22 in her bank account and she deposits $11.50 each week. Janine has $118 in her bank account and she deposits $6.70 each week. In how many weeks, will Lisa have more money in her bank account?

In this problem, we know information about Lisa and Janine’s bank accounts. Therefore, we can write an equation based on Lisa’s account and an equation based on Janine’s account.

For each account we know how much they started with and how much they deposit each week. The amount that they start with is a constant, it doesn’t change. Therefore, it is the y-intercept in the equation. The amount that is deposited is the rate at which the bank balance changes, therefore, it is the slope in the equation. Since they are depositing money, we will have a positive slope. Since I know the slope and y-intercept for each equation, I can write both equations in slope intercept form.

Let \( y \) = The total account balance. \( x \) = the number of weeks

Lisa’s account: \( y = 11.50x + 22 \)  
Janine’s account: \( y = 6.70x + 118 \)

We know that initially Janine has more money in her account because she starts out with more ($118), but eventually Lisa is going to catch up and have the same amount and then she will eventually pass Janine and have more in her account. Lisa will have less until they reach the point where the account balances are the same. This is the solution to the system. Any week thereafter, Lisa will have more money in her account than Janine.

So, let’s solve the system of equations to find the number of weeks where they will have the same account balance and then we’ll know that after this amount of weeks, Lisa will continue to have more money.

Since my equations are both written in slope intercept form, I am going to use the substitution method to solve.

\[
\begin{align*}
y &= 11.50x + 22 \\
y &= 6.70x + 118
\end{align*}
\]

\[
\begin{align*}
6.70x + 118 &= 11.50x + 22 \\
6.70x - 6.70x + 118 &= 11.50x - 6.70x + 22 \\
118 &= 4.8x + 22 \\
118 - 22 &= 4.8x + 22 - 22 \\
96 &= 4.8x \\
96 &= 4.8x \\
4.8x &= 4.8x
\end{align*}
\]

\[x = 20\]

At 20 weeks, the account balances will be the same. **AFTER 20 weeks**, Lisa will begin to have more money in her account. Check: Lisa: \( y = 11.5(21) + 22 \)  

\[ Y = 263.50 \]

Janine: \( y = 6.7x(21) + 118 \)  

\[ y = 258.70 \]

A. 20  
B. 21  
C. 19  
D. 22
5. The Lakers scored a total of 80 points in a basketball game. The Lakers made a total of 37 two-point and three-point baskets.

- Write a linear system of equations that represents this situation.
- How many two-point shots did the Lakers make? Justify your answer.
- How many three-point shots did the Lakers make? Justify your answer.

In this problem we know how many total points were scored and how many baskets were made in the basketball game. Therefore, we can write 2 equations. One equation based on how many points were scored and the other equation based on how many baskets were made.

Since we do not have a rate or a constant, we are going to write our equations in standard form.

Let \( x \) = number of 2 point shots

Let \( y \) = number of 3 point shots

Points: \( 2x + 3y = 80 \)

Number of baskets: \( x + y = 37 \)

In order to find the number of 2 point and 3 point baskets, we need to solve for \( x \) and \( y \). Therefore, we’ll need to solve the system of equations.

I am going to use linear combinations to solve the system since they are both written in standard form.

\[
\begin{align*}
2x + 3y &= 80 \\
-2x - 2y &= -74
\end{align*}
\]

Multiply by -2 to get opposite terms.

\[ y = 6 \]

The \( y \) coordinate of the system is 6.

Substitute 6 for \( y \) into either equation.

\[
\begin{align*}
x + y &= 37 \\
x + 6 &= 37
\end{align*}
\]

Substitute 6 for \( y \).

\[
\begin{align*}
x + 6 - 6 &= 37 - 6 \\
x &= 31
\end{align*}
\]

The \( x \) coordinate is 31.

The solution to this system of equations is \((31, 6)\).
The Lakers made 31 two point shots and 6 three point shots!

Justify: (Substitute into both equations)

\[
\begin{align*}
2x + 3y &= 80 \\
2(31) + 3(6) &= 80
\end{align*}
\]

\[
\begin{align*}
x + y &= 37 \\
31 + 6 &= 37 \quad \text{ yay!}
\end{align*}
\]

\[
\begin{align*}
80 &= 80 \quad \text{ yay!}
\end{align*}
\]
6. The perimeter of an ice skating rink is 180 feet. The length of the rink is 10 feet longer than the width.

- Write a linear system of equations that represents this situation.
- What is the length and width of the ice skating rink? Explain how you determined your answer.

We are given information about the perimeter of the ice rink. In order to begin the problem, you must know (from previous experience) that the perimeter formula for a rectangle is: \( P = 2L + 2w \). (2 times the length + 2 times the width equals the perimeter).

System of Equations:

Let \( w \) = width  
Let \( L \) = length

\[ 180 = 2L + 2w \]  
(We know that 180 is the perimeter of the rink, so substitute 180 for \( P \)).

\[ L = w + 10 \]  
(We know the length is 10 feet longer than the width)

Given this problem, I would use the substitution method because I can substitute \( w + 10 \) for \( l \) into the first equation and solve for \( w \). Once I find \( w \), I can easily substitute to find \( l \).

\[ 180 = 2L + 2w \]

\[ 180 = 2(w+10) + 2w \]  
Substitute \((w+10)\) for \( l \).

\[ 180 = 2w + 20 + 2w \]  
Distribute the 2 throughout the parenthesis.

\[ 180 = 4w + 20 \]  
Combine like terms. \((2w + 2w = 4w)\)

\[ 180 - 20 = 4w + 20 - 20 \]  
Subtract 20 from both sides.

\[ 160 = 4w \]  
Simplify: \(180 - 20 = 160\)

\[ \frac{160}{4} = \frac{4w}{4} \]  
Divide by 4 on both sides.

\[ 40 = w \]  
Simplify: \(160/4 = 40\)

\[ \text{W} = 40 \quad \text{L} = \text{w} + 10 \]

\[ \text{L} = 40 + 10 \]

\[ \text{L} = 50 \]  
\( \text{The width of the ice skating rink is 40 feet. The length is 50 feet.} \)

Explanation: I used the perimeter formula to write my first equation. I knew that the perimeter was 180, so I substituted 180 for \( P \) into the equation: \( P = 2L + 2w \). My second equation was written about the length. Since the length was 10 feet longer than the width, my second equation was: \( l = w + 10 \). I solved the system of equations using the substitution method. I substituted \( w + 10 \) for \( l \) and solved for \( w \). I found \( w \) to equal 40. Since the length is 10 feet longer than the width, the length is 50 feet. \((40 + 10 = 50)\).
7. John is thinking of buying one of two cars. Car A will cost $17655. Average cost per year for fuel, maintenance, and repairs is $1230. Car B will cost about $15900. Fuel, maintenance, and repairs will cost $1425 per year.

- Write a system of equations to represent this situation.
- You expect to only keep the car about 5 years. Which car should you buy? Justify your answer.

I am going to write a system of equations based on the information for car A and car B. I notice that for both cars, I am given a rate (cost per year) and a constant (total cost of the car). Therefore, I can write both equations in slope intercept form.

Let x = # of years   Let y = total cost

Car A:  \( y = 1230x + 17655 \)

Car B:  \( y = 1425x + 15900 \)

I do not need to solve the system of equations because the 2nd bullet tells us that we are only going to keep the car for 5 years. (I don’t need to find the point where the two cars will cost the same amount). Since I know the number of years, I can substitute 5 into each equation and compare.

Car A:  \( Y = 1230(5) + 17655 \)
        \( Y = 6150 + 17655 \)
        \( Y = 23805 \)

Car B:  \( y = 1425(5) + 15900 \)
        \( y = 7125 + 15900 \)
        \( y = 23025 \)

Car A will cost $23805 over 5 years. Car B will cost $23025 over 5 years.

If John buys Car B, he will save money over 5 years. Car B will cost $23025 and Car A will cost $23805.
8. Go Baby Go Inc. manufactures four-wheeled strollers and six-wheeled strollers. The company has 1800 wheels in stock and needs to make a total of 350 strollers.

- Write a system of equations that represents this situation.
- How many of each product can the company make? Justify your answer.

We are given information about 4 wheeled strollers and 6 wheeled strollers. We know that there are 1800 wheels total and they need to make 350 strollers. I need to write 2 equations, one based on the number of strollers and one based on the number of wheels.

Let \( x \) = the number of 4 wheeled strollers  
Let \( y \) = the number of 6 wheeled strollers

\[
\begin{align*}
\text{\( x + y = 350 \)} & \quad \text{(Equation based on number of strollers. \# of 4 wheeled + \# of 6 wheeled = 350)} \\
\text{\( 4x + 6y = 1800 \)} & \quad \text{(Equation based on number of wheels. Each 4 wheeled stroller requires 4 wheels (4x) and each 6 wheeled stroller requires 6 wheels (6y) and the total number of wheels is 1800.)}
\end{align*}
\]

In order to find out how many of each product the company can make, I will need to solve for \( x \) and \( y \). Therefore, I need to solve the system of equations. Since both equations are written in standard form, I am going to use linear combinations as my method for solving.

\[
\begin{align*}
\text{-4(x +y = 350)} & \quad \text{-4x -4y = -1400} & \quad \text{Multiply all terms by -4 to create opposite x terms.} \\
\text{4x + 6y = 1800} & \quad \text{Keep this equation the same.} \\
2y & = 400 \quad \text{Add like terms in the 2 equations.} \\
2y & = 400 \quad \text{Divide both terms by 2.} \\
\text{Y = 200} & \quad \text{y = 200} \quad \text{The number of 6 wheeled strollers is 200.}
\end{align*}
\]

\[
\begin{align*}
\text{x + y = 350} \\
\text{x + 200 = 350} \quad \text{Substitute 200 for y into the original equation.} \\
\text{x +200 -200 = 350 -200} \quad \text{Subtract 200 from both sides.} \\
\text{x = 150} \quad \text{The number of 4 wheeled strollers is 150}
\end{align*}
\]

The company is able to make 200 6-wheeled strollers and 150 4-wheeled strollers.

Justify: 
\[
\begin{align*}
\text{x + y = 350} & \quad \text{4x + 6y = 1800} \\
\text{150 + 200 = 350} & \quad \text{4(150) + 6(200) = 1800} \\
\text{350 = 350} \quad \text{600 + 1200 = 1800} \\
\quad \text{1800 = 1800}
\end{align*}
\]
9. The local high school is selling pizza kits to raise money for the end of the year field trip. The high school pays $30 for a contract with the pizza company and $5 for each kit. The selling price of each kit is $9.

- Write a system of equations to represent this situation.
- How many kits must the high school sell in order to break even? Justify your answer.

We are given information about how much the school pays the pizza company and how much the school charges for each kit. I need to write 2 equations, one based on how much the school pays and one on how much the school charges. The information about how much the school pays shows that I have a rate ($5 for each kit) and a constant ($30 flat for the contract). Therefore I can write the equation in slope intercept form.

Let \( x \) = # of kits ordered 
Let \( y \) = total price

\[
\begin{align*}
  y &= 5x + 30 \quad \text{(The equation shows how much the school pays for each the pizza kits)} \\
  y &= 9x \quad \text{(This equation shows how much the school makes for each kit)}
\end{align*}
\]

Since I need to find the number of kits that they must sell in order to break even, I must find the point of intersection. Therefore, I need to solve the system of equations. I am going to use the substitution method since both equations are written in slope intercept form.

\[
\begin{align*}
  y &= 5x + 30 \\
  y &= 9x \\
  9x &= 5x + 30 \quad \text{Substitute 9x for y into the first equation.} \\
  9x - 5x &= 5x -5x +30 \quad \text{Subtract 5x from both sides.} \\
  4x &= 30 \quad \text{Simplify: } (9x – 5x = 4x) \\
  4x &= 30 \quad \text{Divide both sides by 4} \\
  x &= 7.5
\end{align*}
\]

The high school must sell 7.5 kits to break even. Since you can’t sell half of a kit, the high school must sell 8 kits in order to break even. If they sell more, they will make a profit.

Justify:
\[
\begin{align*}
  y &= 5x + 30 \\
  y &= 9x \\
  Y &= 5(7.5) +30 \quad y=9(7.5) \\
  Y &= 67.5 \quad 67.5
\end{align*}
\]

Both equations are equal to 67.5 when you substitute 7.5 for \( x \). Therefore, I know my answer is correct. However, since this is a real world situation and we know that we can’t sell \( \frac{1}{2} \) of a pizza kit, the correct answer is that they must sell 8 kits in order to break even.
10. Jerry wants to print a bunch of old pictures. He has decided upon two companies than can print the pictures for him.

<table>
<thead>
<tr>
<th>Company</th>
<th>Membership Fee</th>
<th>Price per photo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photome.com</td>
<td>$10</td>
<td>$0.10</td>
</tr>
<tr>
<td>Smile.com</td>
<td>$15</td>
<td>$0.06</td>
</tr>
</tbody>
</table>

• Write an equation to represent the cost for Photome.com.
• Write an equation to represent the cost for Smile.com.
• Jerry plans to print 200 pictures. Which company should he choose? Justify your answer.

I am given information about 2 different companies: Photome.com and Smile.com. For each company I am given the membership fee and the price per photo. Therefore, I am given a rate (price per photo) and a constant (membership fee). I can write both equations in slope intercept form.

Let \( x \) = the number of photos

Let \( y \) = total cost

**Photome.com**  
\[ y = 0.10x + 10 \]

**Smile.com**  
\[ y = 0.06x + 15 \]

Since I know that I need to find the price for 200 pictures, I don’t need to solve the system of equations. I can substitute 200 for \( x \) into both equations and compare the results.

**Photome.com**  
\[ Y = 0.10(200) + 10 \]
\[ Y = 30 \]

**Smile.com**  
\[ y = 0.06(200) + 15 \]
\[ y = 27 \]

Jerry should choose Smile.com to print his pictures because his total cost would be $27 versus $30 for Photome.com. He would save money by going with Smile.com for developing 200 pictures.
1. Missy has $200 and saves $50 a week. Marcus has $1800 and spends $50 a week. How many weeks from now will they have the same amount of money? How much money will they each have? (3 points)

You must first write an equation to match Missy’s savings and Marcus’s savings.

Missy: \( y = 50x + 200 \) (Saves $50 per week and she has $200. 200 is the constant (y-intercept) and 50 is the rate: key word: per)

Marcus: \( y = 1800 - 50x \) (Spends $50 per week, so we use the subtraction and 1800 is the constant because that is what he starts with.)

To find out when they will have the same amount of money, we must solve this system of equations. Since both equations are written in slope intercept form and already solved for \( y \), I will use the substitution method.

\[
50x + 200 = 1800 - 50x \\
50x + 50x + 200 = 1800 - 50x + 50x \quad \text{Add 50x to both sides in order to solve for } x. \\
100x + 200 = 1800 \quad \text{Simplify: } 50x + 50x = 100x \\
100x + 200 - 200 = 1800 - 200 \quad \text{Subtract 200 from both sides} \\
100x = 1600 \quad \text{Simplify: } 1800 - 200 = 1600 \\
100x/100 = 1600/100 \quad \text{Divide by 100 on both sides} \\
X = 16 \quad x = 160, \text{ so after 16 weeks, they will have the same amount.}
\]

In order to figure out how much they will have after 16 weeks, we need to substitute 16 for \( x \) into each equation:

Missy: \( y = 50x + 200 \) 
\[
Y = 50(16) + 200 \\
Y = 1000
\]

Marcus: \( y = 1800 - 50x \) 
\[
y = 1800 - 50(16) \\
y = 1000
\]

After 16 weeks, they will have the same amount of $1000.
2. A resort hotel is offering a special for the holiday weekend:
   Special #1: two nights and 1 meal for two for $215
   Special #2: three nights and 2 meals for two for $345

   What price is the hotel charging per night and per meal? (3 points)

   We must write a system of equation and solve in order to find the price that the hotel is charging and the price per meal.

   Let x = price per night   Let y = the price per meal

   Special #1: 2x + y = 215  (2 nights·price + 1 meal· price = 215)
   Special #2: 3x + 2y = 345  (3 nights·price+ 2 meals· price = 345)

   Now, that I have written a system of equations, I can solve for x and y which will tell me the price of the hotel and the price of the meal.

   I don’t have opposite terms, so I will multiply equation #1 by -2 in order to create opposite y terms.

   -2(2x + y) = -2 (215)  \rightarrow -4x -2y = -430  \quad \text{Multiply by -2}
   3x+2y = 345  \quad \text{Keep the same}
   -x = -85  \quad \text{Add the equations}

   -x = - 85  \quad \text{Now we will solve for x.}
   (-1)-x = -85(-1)  \quad \text{Multiply by -1 in order to make x positive.}
   \textbf{X = 85}  \quad \textbf{x = 85 and now we must solve for y.}

   2x + y = 215  \quad \text{I will use this equation to substitute 85 for x and solve for y.}
   2(85) + y = 215  \quad \text{Substitute 85 for x.}
   170 +y = 215  \quad \text{Simplify: 2(85) = 170}
   170-170 + y = 215-170  \quad \text{Subtract 170 from both sides}
   \textbf{Y = 45}  \quad \text{Simplify: 215-170 = 45}

   Since \textbf{x = 85}, and \textbf{x} is the price of the hotel per night, we know that the hotel cost is $85 per night.
   Since \textbf{y = 45} and this is the price of a meal, each meal for two costs $45.
3. Jesse’s results on a standardized test show that his math score was 70 points higher than his reading score. His total score for the two parts is 1340. Let \( m = \) math score and \( r = \) reading score. (4 points)

- Write a system of equations for this situation.

Let \( m = \) math score and \( r = \) reading score.

We know that Jesse’s math score was 70 points higher than reading, so: \( m = r + 70 \)

We know that his total score (when adding both parts together) was 1340, so: \( m + r = 1340 \)

**Our system of equations for this situation is:**

\[
\begin{align*}
\text{m} &= \text{r} + 70 \\
\text{m} + \text{r} &= 1340
\end{align*}
\]

- Find Jesse’s math score and reading score.

In order to find Jesse’s math and reading score, we must solve the system of equations.

One equation is written in standard form, and the other is solved for \( m \). This means that the substitution method would be the best choice because we have one equation solved for \( m \).

Let’s substitute \( r + 70 \) for \( m \) into the second equation.

\[
\begin{align*}
\text{m} + \text{r} &= 1340 \quad \text{The second equation} \\
\text{r} + 70 + \text{r} &= 1340 \quad \text{Substitute } r + 70 \text{ for } m. \\
2\text{r} + 70 &= 1340 \quad \text{Combine like terms: } (r + r = 2r) \\
2\text{r} + 70 - 70 &= 1340 - 70 \quad \text{Subtract 70 from both sides.} \\
2\text{r} &= 1270 \quad \text{Simplify: } 1340 - 70 = 1270 \\
\frac{2\text{r}}{2} &= \frac{1270}{2} \quad \text{Divide by } 2 \text{ on both sides} \\
\text{r} &= 635
\end{align*}
\]

Now we know \( r = 635 \), we must find \( m \) by substituting 635 for \( r \) back into the first equation.

\[
\begin{align*}
\text{m} &= \text{r} + 70 \\
\text{m} &= 635 + 70 \\
\text{m} &= 705
\end{align*}
\]

**Jesses’ math score was 705 and Jesse’s reading score was 635.**

- Justify your answers using mathematics.

Justify by substituting 705 and 635 back into both equations:

\[
\begin{align*}
\text{m} &= \text{r} + 70 \\
705 &= 635 + 70 \\
705 &= 705
\end{align*}
\]

\[
\begin{align*}
\text{m} + \text{r} &= 1340 \\
705 + 635 &= 1340 \\
1340 &= 1340
\end{align*}
\]

Since both equations are mathematically correct, we know that your solution is correct.
Lesson 5: Solving Systems in Three Variables

An example of a system of equations in 3 variables is:

_____________________________
_____________________________
_____________________________

The graph of a system of equations in 3 variables, all to the first power is a ______________.

The solution can be one point, infinite (in the same plane (or a line) and no solution with no points in common.

In this picture, we see 3 planes that intersect in a line.

Planes that intersect in a line, in the same plane, have an ________________________ number of solutions.

These 3 planes have no point in common. Therefore, there are ________________________.
Example 1 – A System with One Solution

Solve the system of equations: \( x+y+z = 2 \) \( x+2y-z = 6 \) \( 2x+y-z = 5 \)

**Step 1:** Use elimination to write one of the equations in 2 variables.

**Step 2:** Repeat step 1 with 2 different equations. (One equation will be used again). You must also eliminate the same variable as you eliminated in step 1.

**Step 3:** Solve the system of equations with 2 variables (Equations 4 & 5)

**Step 4:** Substitute your answer to step 3 into one of the 2 variable equations and solve.

**Step 5:** Substitute your values from step 3 and 4 into 1 of the original 3 variable equations and solve for the missing variable.

**Step 6:** Solution:
### Example 2: A System with an Infinite Number of Solutions

Solve the system of equations:

\[
\begin{align*}
3x - 2y + 3z &= 8 \\
x + 3y + 6z &= -3 \\
2x + 6y + 12z &= -6
\end{align*}
\]

**Step 1:** Use elimination to write one of the equations in 2 variables.

**Step 2:** Repeat step 1 with 2 different equations. (One equation will be used again). You must also eliminate the same variable as you eliminated in step 1.

**Step 3:** Solve the system of equations with 2 variables (Equations 4 & 5)

**Step 4:** Substitute you answer to step 3 into one of the 2 variable equations and solve.

**Step 5:** Substitute your values from step 3 and 4 into 1 of the original 3 variable equations and solve for the missing variable.

**Step 6:** Solution:
Example 3: A System with No Solutions

Solve the system of equations: \[2x + 4y + 10z = 14\] \[x + 2y + 5z = -4\] \[3x - 4y - 3z = 15\]

Step 1: Use elimination to write one of the equations in 2 variables.

Step 2: Repeat step 1 with 2 different equations. (One equation will be used again). You must also eliminate the same variable as you eliminated in step 1.

Step 3: Solve the system of equations with 2 variables (Equations 4 & 5)

Step 4: Substitute your answer to step 3 into one of the 2 variable equations and solve.

Step 5: Substitute your values from step 3 and 4 into 1 of the original 3 variable equations and solve for the missing variable.

Step 6: Solution:
Example 4: Real World Applications

A bakery sells three types of donuts. Jelly filled donuts cost $0.45 a piece. Frosted donuts cost $0.40 a piece and plain donuts cost $0.30 a piece. Joe bought 22 donuts and paid $8.00. He bought twice as many frosted donuts as jelly filled donuts. How many of each type of donut did Joe buy?
Lesson 5: Solving Systems in Three Variables – Practice Problems

Part 1: Solve each system of equations.

1. \[4x - 3y + z = -10\]  \[2x + y + 3z = 0\]  \[-x + 2y - 5z = 17\]

2. \[4x - 2y + 3z = 1\]  \[x + 3y - 4z = 7\]  \[3x + y + 2z = 5\]

3. \[x + y - z = 5\]  \[3x - 2y + z = 8\]  \[2x + 2y - 2z = 7\]

4. \[3x - 2y + 4z = 20\]  \[-x + 5y + 12z = 73\]  \[x + 3y - 2z = 1\]

5. \[2x - 3y - z = 4\]  \[-6x + 9y + 3z = -12\]  \[4x - 6y - 2z = 8\]

6. \[x + 2y - z = 4\]  \[3x - y + z = 5\]  \[2x + 3y + 2z = 7\]

7. \[2x - y + z = 10\]  \[3x - 2y - 2z = 7\]  \[x - 3y - 2z = 10\]

8. \[x - 3y + 2z = -11\]  \[2x - 4y + 3z = -15\]  \[3x - 5y - 4z = 5\]

9. \[x + y + z = 1\]  \[2x + 2y + 2z = 2\]  \[4x + 4y + 4z = 4\]

10. \[-2x + 3y - 4z = 3\]  \[3x - 5y + 2z = 4\]  \[-4x + 2y - 3z = 0\]
Part 2: Write a system of equations and then solve.

11. The sum of 3 numbers is 14. The second number is twice the first and the sum of the first and third numbers is 6. Find the numbers.

12. The sum of three numbers is -28. The second number is half of the first number. The sum of the first and third number is -24. Find the numbers.

13. Joe went to the bank to get cash to take on a trip. He received bills in the denominations of $20, $50, and $100. His withdrawal totaled $620. The number of $20 dollar bills is 3 times the amount of $50 dollar bills. The 50 dollar bills and $100 bills add up to $500. How many of each type of bill did Joe receive?

14. Mike has 19 coins (nickels, dimes and quarters) in his piggy bank with a total value of $2.35. There are 2 more dimes than nickels. How many of each type does Mike have?

JoAnne sold magazine subscriptions with three prices: $20, $17, and $15. She sold 2 fewer of the $20 subscriptions than of the $17 subscription and sold a total of 27 subscriptions. If her total sales amounted to $479, how many of each did she sell?
Lesson 5: Solving Systems in Three Variables – Practice Problems – Answer Key

Part 1: Solve each system of equations.

1. \[ 4x - 3y + z = -10 \] \hspace{1cm} 2. \[ 2x + y + 3z = 0 \] \hspace{1cm} 3. \[ -x + 2y - 5z = 17 \]

<table>
<thead>
<tr>
<th>Step</th>
<th>Equation 1</th>
<th>Equation 2</th>
<th>Equation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>[ 4x - 3y + z = -10 ]</td>
<td>[ 4x - 3y + z = -10 ]</td>
<td>[ -10 ]</td>
</tr>
<tr>
<td></td>
<td>[ -2(2x + y + 3z) = 0 ]</td>
<td>[ -4x - 2y - 6z = 0 ]</td>
<td>[ -5y - 5z = -10 ]</td>
</tr>
<tr>
<td>2.</td>
<td>[ 2x + y + 3z = 0 ]</td>
<td>[ 2x + y + 3z = 0 ]</td>
<td>[ 34 ]</td>
</tr>
<tr>
<td></td>
<td>[ 2(-x + 2y - 5z) = 17 ]</td>
<td>[ -2x + 4y - 10z = 34 ]</td>
<td>[ 5y - 7z = 34 ]</td>
</tr>
<tr>
<td>3.</td>
<td>[ -5y - 5z = -10 ]</td>
<td>[ 5y - 7z = 34 ]</td>
<td>[ z = -2 ]</td>
</tr>
<tr>
<td></td>
<td>[ 5y - 7z = 34 ]</td>
<td>[ 5y - 7z = 34 ]</td>
<td>[ y = 4 ]</td>
</tr>
<tr>
<td></td>
<td>[ -12z = 24 ]</td>
<td>[ -12z = 24 ]</td>
<td>[ x = 1 ]</td>
</tr>
<tr>
<td></td>
<td>[ z = -2 ]</td>
<td>[ Solution: (1, 4, -2) ]</td>
<td></td>
</tr>
</tbody>
</table>

Step 4: Substitute -2 for \( z \) into the equation \( 5y - 7z = 34 \)

\[ 5y - 7(-2) = 34 \]
\[ 5y + 14 = 34 \]
\[ 5y = 20 \]
\[ y = 4 \]

Step 5: Substitute \( y = 4 \) and \( z = -2 \) into the equation, \( 2x + y + 3z = 0 \)

\[ 2x + y + 3z = 0 \]
\[ 2x + 4 + 3(-2) = 0 \]
\[ 2x - 2 = 0 \]
\[ 2x - 2 + 2 = 0 + 2 \]
\[ 2x = 2 \]
\[ 2x / 2 = 2 / 2 \]
\[ x = 1 \]

Step 6: Solution: \( (1, 4, -2) \)
2. \[4x - 2y + 3z = 1\] \[x + 3y - 4z = -7\] \[3x + y + 2z = 5\]

**Step 1:** Eliminate \(x\) from Equations 1 & 2

\[
\begin{align*}
4x - 2y + 3z &= 1 \\
-4(x + 3y - 4z) &= -7(-4) \\
\end{align*}
\]

\[-14y + 19z = 29\]

**Step 2:** Eliminate \(x\) from Equations 2 & 3.

\[
\begin{align*}
-3(x + 3y - 4z) &= -7(-3) \\
3x + y + 2z &= 5 \\
\end{align*}
\]

\[-8y + 14z = 26\]

**Step 3:** Solve the System with Equations 4 & 5.

\[
\begin{align*}
-4(-14y + 19z) &= 29(-4) \\
56y - 76z &= -116 \\
7(-8y + 14z) &= -26(7) \\
-56y + 98z &= 182 \\
22z &= 66 \\
z &= 3
\end{align*}
\]

**Step 4:** Substitute 3 for \(z\) into the equation \(-8y + 14z = 26\)

\[
\begin{align*}
-8y + 14z &= 26 \\
-8y + 14(3) &= 26 \\
-8y + 42 &= 26 \\
-8y &= -16 \\
y &= 2
\end{align*}
\]

**Step 5:** Substitute \(y = 2\) and \(z = 3\) into the equation, \(4x - 2y + 3z = 1\)

\[
\begin{align*}
4x - 2y + 3z &= 1 \\
4x - 2(2) + 3(3) &= 1 \\
4x &= 1 \\
4z + 5 &= 1 \\
4z &= -4 \\
z &= -1
\end{align*}
\]

**Step 6:** Solution: \((-1, 2, 3)\)
3. \[ x+y-z=5 \quad \text{3x-2y+z=8} \quad \text{2x+2y-2z=7} \]

**Step 1:** Eliminate x from Equations 1 & 2

\[-3(x+y-z) = 5(-3) \quad \Rightarrow \quad -3x - 3y + 3z = -15\]
\[3x - 2y + z = 8 \quad \Rightarrow \quad 3x - 2y + z = 8\]
\[-5y + 4z = -7\]

**Step 2:** Eliminate x from Equations 1 & 3.

\[-2(x+y-z) = 5(-2) \quad \Rightarrow \quad -2x - 2y + 2z = -10\]
\[2x + 2y - 2z = 7 \quad \Rightarrow \quad 2x + 2y - 2z = 7\]
\[0 = -3\]

Since this statement does not make sense, we can stop here. This means that at least 2 of the planes do not intersect, which means that there is no solution.

4. \[ 3x - 2y + 4z = 20 \quad -x + 5y + 12z = 73 \quad x + 3y - 2z = 1 \]

**Step 1:** Eliminate x from Equations 2 & 3 (already set up)

\[-x + 5y + 12z = 73\]
\[x + 3y - 2z = 1\]
\[8y + 10z = 74\]

**Step 2:** Eliminate x from Equations 1 & 2.

\[3x - 2y + 4z = 20\]
\[3(-x + 5y + 12z) = 73(3) \quad \Rightarrow \quad -3x + 15y + 36z = 219\]
\[13y + 40z = 239\]

**Step 3:** Solve the System with Equations 4 & 5.

\[-4(8y + 10z) = 74(-4) \quad \Rightarrow \quad -32y - 40z = -296\]
\[13y + 40z = 239\]
\[13y + 40z = 239\]
\[-19y = -57\]
\[y = 3\]

**Step 4:** Substitute 3 for y into the equation 8y + 10z = 74

\[8y + 10z = 74\]
\[8(3) + 10z = 74\]
\[24 + 10z = 74\]
\[24 - 24 + 10z = 74 - 24\]
\[10z = 50\]
\[z = 5\]

**Step 5:** Substitute y = 3 and z = 5 into the equation, x + 3y - 2z = 1

\[x + 3y - 2z = 1\]
\[x + 3(3) - 2(5) = 1\]
\[x - 1 = 1\]
\[x = 2\]

Solution: (2, 3, 5)
5. \[2x - 3y - z = 4 \quad -6x + 9y + 3z = -12 \quad 4x - 6y - 2z = 8\]

**Step 1: Eliminate \(x\) from Equations 1 & 2**

\[
\begin{align*}
3(2x - 3y - z) &= 4(3) \\
-6x + 9y + 3z &= -12 \\
0 &= 0
\end{align*}
\]

This is a true statement, therefore, this system could intersect at a line and have an infinite number of solutions. These two planes are the same plane. We need to make sure that the third plane intersects as well. Therefore, let's see what happens when we eliminate \(x\) from equations 1 and 3.

**Step 2: Eliminate \(x\) from Equations 1 & 3.**

\[
\begin{align*}
-2(2x - 3y - z) &= 4(-2) \\
4x - 6y - 2z &= 8 \\
0 &= 0
\end{align*}
\]

Since this also indicates that plane 1 and 3 are the same, this means that all three planes are the same plane. **Therefore, there are an infinite number of solutions.**

**Solution: (infinite solutions)**

- \(y = \) __________
- \(z = 5\)
- \(x = \) __________
6. \(x + 2y - z = 4\) \(3x - y + z = 5\) \(2x + 3y + 2z = 7\)

**Step 1:** Eliminate \(z\) from Equations 1 & 2 (already set up)

\[
\begin{align*}
x + 2y - z &= 4 \\
3x - y + z &= 5 \\
4x + y &= 9
\end{align*}
\]

**Step 2:** Eliminate \(z\) from Equations 1 & 3.

\[
\begin{align*}
2(x + 2y - z) &= 4(2) \\
2x + 3y + 2z &= 7
\end{align*}
\]

\[
\begin{align*}
4x + y &= 9 \\
4x + 7y &= 15
\end{align*}
\]

**Step 3:** Solve the System with Equations 4 & 5.

\[
\begin{align*}
-1(4x + y) &= 9(-1) \\
-4x - y &= -9 \\
4x + 7y &= 15
\end{align*}
\]

\[
\begin{align*}
6y &= 6 \\
y &= 1
\end{align*}
\]

**Step 4:** Substitute 1 for \(y\) into the equation \(4x + y = 9\)

\[
\begin{align*}
4x + y &= 9 \\
4x + 1 &= 9 \\
4x + 1 - 1 &= 9 - 1 \\
4x &= 8 \\
x &= 2
\end{align*}
\]

**Step 5:** Substitute \(y = 1\) and \(x = 2\) into the equation, \(x + 2y - z = 4\)

\[
\begin{align*}
x + 2y - z &= 4 \\
2 + 2(1) - z &= 4 \\
4 - z &= 4 \\
4 - 4 - z &= 4 - 4 \\
-z &= 0 \text{ and } z = 0
\end{align*}
\]

**Step 6:** Solution: \((2, 1, 0)\)
7. \(2x - y + z = 10\) \quad \(3x - 2y - 2z = 7\) \quad \(x - 3y - 2z = 10\)

**Step 1: Eliminate \(y\) from Equations 1 \& 2**

\[
\begin{align*}
-2(2x - y + z) &= 10(-2) \\
3x - 2y - 2z &= 7
\end{align*}
\]

\[
\begin{align*}
-4x + 2y - 2z &= -20 \\
3x - 2y - 2z &= 7 \\
-x - 4z &= -13
\end{align*}
\]

**Step 2: Eliminate \(y\) from Equations 1 \& 3.**

\[
\begin{align*}
-3(2x - y + z) &= 10(-3) \\
x - 3y - 2z &= 10
\end{align*}
\]

\[
\begin{align*}
-6x + 3y - 3z &= -30 \\
x - 3y - 2z &= 10 \\
-5x - 5z &= -20
\end{align*}
\]

**Step 3: Solve the System with Equations 4 \& 5.**

\[
\begin{align*}
-5(x - 4z) &= -13(-5) \\
5x + 20z &= 65 \\
-5x - 5z &= -20
\end{align*}
\]

\[
\begin{align*}
x - 4z &= -13 \\
-x - 4(3) &= -13 \\
-x - 12 &= -13 \\
-x - 12 + 12 &= -13 + 12 \\
-x &= -1 \\
x &= 1
\end{align*}
\]

**Step 4: Substitute 3 for \(z\) into the equation \(-x - 4z = -13\)**

\[
\begin{align*}
-x - 4z &= -13 \\
-x - 4(3) &= -13 \\
-x - 12 &= -13 \\
-x - 12 + 12 &= -13 + 12 \\
-x &= -1 \\
x &= 1
\end{align*}
\]

**Step 5: Substitute \(z = 3\) and \(x = 1\) into the equation, \(2x - y + z = 10\)**

\[
\begin{align*}
2x - y + z &= 10 \\
2(1) - y + 3 &= 10 \\
5 - y &= 10 \\
5 - 5 - y &= 10 - 5 \\
-y &= 5 \\
y &= -5
\end{align*}
\]

**Step 6: Solution: \((1, -5, 3)\)**
### Step 1: Eliminate $x$ from Equations 1 & 2

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 3y + 2z = -11$</td>
<td>$2x - 4y + 3z = -15$</td>
</tr>
</tbody>
</table>

\[-2(x - 3y + 2z) = -11(-2)\]
\[-2x + 6y - 4z = 22\]
\[2x - 4y + 3z = -15\]
\[2y - z = 7\]

### Step 2: Eliminate $x$ from Equations 1 & 3.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x - 3y + 2z = -11$</td>
<td>$3x - 5y - 4z = 5$</td>
</tr>
</tbody>
</table>

\[-3(x - 3y + 2z) = -11(-3)\]
\[-3x + 9y - 6z = 33\]
\[3x - 5y - 4z = 5\]
\[4y - 10z = 38\]

### Step 3: Solve the System with Equations 4 & 5.

<table>
<thead>
<tr>
<th>Equation 4</th>
<th>Equation 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2y - z = 7$</td>
<td>$4y - 10z = 38$</td>
</tr>
</tbody>
</table>

\[-2(2y - z) = 7(-2)\]
\[-4y + 2z = -14\]
\[4y - 10z = 38\]
\[-8z = 24\]
\[z = -3\]

### Step 4: Substitute $z = -3$ into the equation $2y - z = 7$

\[2y - (-3) = 7\]
\[2y + 3 = 7\]
\[2y = 4\]
\[y = 2\]

### Step 5: Substitute $z = -3$ and $y = 2$ into the equation, $x - 3y + 2z = -11$

\[x - 3y + 2z = -11\]
\[x - 3(2) + 2(-3) = -11\]
\[x - 12 = -11\]
\[x - 12 + 12 = -11 + 12\]
\[x = 1\]

### Step 6: Solution: $(1, 2, -3)$
**You may be able to tell from the equations that these are all the same plane. Equations 2 and 3 are multiples of equation 1. Thus, we can say that the solution is infinite set of points because they are all the same plane.**

<table>
<thead>
<tr>
<th>Step 1: Eliminate x from Equations 1 &amp; 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(-2x +3y -4z) = 3(3)</td>
</tr>
<tr>
<td>2(3x – 5y +2z) = 4(2)</td>
</tr>
<tr>
<td>-6x +9y -12z = 9</td>
</tr>
<tr>
<td>6x-10y +4z = 8</td>
</tr>
<tr>
<td>-y -8z = 17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 2: Eliminate x from Equations 1 &amp; 3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2(-2x +3y -4z) = 3(-2)</td>
</tr>
<tr>
<td>-4x +2y – 3z = 0</td>
</tr>
<tr>
<td>-4x +2y -3z = 0</td>
</tr>
<tr>
<td>-4y +5z = -6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 3: Solve the System with Equations 4 &amp; 5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4(-y – 8z) = 17(-4)</td>
</tr>
<tr>
<td>-4y +5z = -6</td>
</tr>
<tr>
<td>4y +32z = -68</td>
</tr>
<tr>
<td>37z = -74</td>
</tr>
<tr>
<td>z = -2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 4: Substitute -2 for z into the equation -y -8z = 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>-y -8z = 17</td>
</tr>
<tr>
<td>-y – 8(-2) = 17</td>
</tr>
<tr>
<td>-y +16 = 17</td>
</tr>
<tr>
<td>-y +16-16 = 17-16</td>
</tr>
<tr>
<td>-y = 1 y = -1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 5: Substitute z = -2 and y = -1 into the equation, -4x +2y – 3z = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4x +2y – 3z = 0</td>
</tr>
<tr>
<td>-4x+2(-1) -3(-2) = 0</td>
</tr>
<tr>
<td>-4x+4 = 0</td>
</tr>
<tr>
<td>-4x +4-4 = 0-4</td>
</tr>
<tr>
<td>-4x = -4</td>
</tr>
<tr>
<td>x = 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Step 6: Solution: (1, -1, -2)</th>
</tr>
</thead>
</table>
Part 2: Write a system of equations and then solve.

11. The sum of 3 numbers is 14. The second number is twice the first and the sum of the first and third numbers is 6. Find the numbers.

**Define your variables and write the system of equations:**

Let \( x \) = the first number  
Let \( y \) = the second number  
Let \( z \) = the third number  

1. \( x+y+z = 14 \)  
   (the sum of 3 numbers is 14)  
2. \( y = 2x \)  
   (the second number is twice the first)  
3. \( x +z = 6 \)  
   (the sum of the first and third number is 6)  

**Step 1: Use substitution to eliminate the \( y \) variable:** Substitute \( y = 2x \) into \( x+y+z = 14 \)

\[
x + 2x +z = 14 \quad \text{(Substitute 2x for y into the equation.)}
\]

\[
3x +z = 14 \quad \text{(Combine like terms)}
\]

**Step 2: Solve the system using the 4^{th} and 3^{rd} Equation.**

\[
3x +z = 14
\]
\[
-1(x +z) = 6(-1)
\]
\[
2x = 8
\]
\[
x = 4
\]

**Step 3: Use \( x = 4 \) with the equation \( y = 2x \), in order to solve for \( y \).**

\[
Y = 2x
\]
\[
Y = 2(4) \quad \text{(Substitute 4 for x.)}
\]
\[
Y = 8
\]

**Step 4: Substitute \( x = 4 \) into the equation \( x+z =6 \) to solve for \( z \).**

\[
x+z = 6
\]
\[
4 +z = 6
\]
\[
4-4 +z = 6 - 4
\]
\[
z = 2
\]

The first number is 4, the second number is 8, and the third number is 2.
12. The sum of three numbers is -28. The second number is half of the first number. The sum of the first and third number is -24. Find the numbers.

**Define your variables and write the system of equations:**

Let \(x\) = the first number
Let \(y\) = the second number
Let \(z\) = the third number

1. \(x+y+z = -28\) 
   (the sum of 3 numbers is -28)
2. \(y = \frac{1}{2}x\) 
   (the second number is half of the first)
3. \(x +z = -24\) 
   (the sum of the first and third number is -24)

**Step 1: Use substitution to eliminate the \(y\) variable:** Substitute \(y = \frac{1}{2}x\) into \(x+y+z = -28\)

\[
x + \frac{1}{2}x +z = -28
\]

Combine like terms

\[
\frac{3}{2}x +z = -28
\]

**Step 2: Solve the system using the 4\(^{th}\) and 3\(^{rd}\) Equation.**

\[
\frac{3}{2}x + z = -28
\]

Substitute \( \frac{1}{2}x\) for \(y\) into the equation.

\[
-x -z = 24
\]

\[
\frac{1}{2}x = -4
\]

\[
x = -8
\]

**Step 3: Use \(x = -8\) with the equation \(y = \frac{1}{2}x\), in order to solve for \(y\).**

\[
Y = \frac{1}{2}x
\]

\[
Y = \frac{1}{2}(-8) \quad \text{Substitute } -8 \text{ for } x.
\]

\[
Y = -4
\]

**Step 4: Substitute \(x = -8\) into the equation \(x+z = -24\) to solve for \(z\).**

\[
x+z = -24
\]

\[
-8 +z = -24
\]

\[
-8+8 +z = -24+8
\]

\[
z = -16
\]

**Solution:** The first number is -8, the second number is -4, and the third number is -16.
13. Joe went to the bank to get cash to take on a trip. He received bills in the denominations of $20, $50, and $100. His withdrawal totaled $620. The number of $20 dollar bills is 3 times the amount of $50 dollar bills. The $50 dollar bills and $100 bills add up to $500. How many of each type of bill did Joe receive?

**Define your variables and write the system of equations:**

Let \(x\) = number of $20 bills  
Let \(y\) = number of $50 bills  
Let \(z\) = number of $100 bills

1. \(20x + 50y + 100z = 620\) (His withdrawal totaled $620)
2. \(x = 3y\) (The number of $20 bills is 3 times the amount of $50 bills)
3. \(50y + 100z = 500\) (The $50 bills and $100 bills add up to $500)

**Step 1:** Use substitution to eliminate the \(x\) variable: Substitute \(x = 3y\) into \(20x + 50y + 100z = 620\)

\[
20x + 50y + 100z = 620
\]

\[
20(3y) + 50y + 100z = 620 \quad \text{Substitute } 3y \text{ for } x \text{ into the equation.}
\]

\[
60y + 50y + 100z = 620
\]

\[
110y + 100z = 620 \quad \text{Combine like terms}
\]

**Step 2:** Solve the system using the 4\(^{th}\) and 3\(^{rd}\) Equation.

\[
110y + 100z = 620
\]

\[
-1(50y + 100z) = 500(-1)
\]

\[
-50y - 100z = -500
\]

\[
60y = 120
\]

\[
y = 2
\]

**Eq 4: 110y + 100z = 620**

\[
x = 6
\]

\[
y = 2
\]

\[
z = 4
\]

**Step 3:** Use \(y = 2\) with the equation \(x = 3y\), in order to solve for \(x\).

\[
x = 3y
\]

\[
x = 3(2)
\]

\[
x = 6
\]

**Step 4:** Substitute \(y = 2\) into the equation, \(50y + 100z = 500\) in order to solve for \(x\).

\[
50y + 100z = 500
\]

\[
50(2) + 100z = 500
\]

\[
100 + 100z = 500
\]

\[
100 - 100 + 100z = 500 - 100
\]

\[
100z = 400
\]

\[
z = 4
\]

**Solution:** Joe received 6 twenty dollar bills, 2 fifty dollar bills, and 4 one-hundred dollar bills.
14. Mike has 19 coins (nickels, dimes and quarters) in his piggy bank with a total value of $2.35. There are 2 more dimes than nickels. How many of each type does Mike have?

**Define your variables and write the system of equations:**

Let \( x \) = number of nickels  
Let \( y \) = number of dimes  
Let \( z \) = number of quarters

(In order to eliminate working with decimals, you can multiply all numbers by 100)

1. \( 5x +10y+25z = 235 \)  \( \) (The total value is $2.35  \( \) multiplied by 100= 235)
2. \( y = x +2 \)  \( \) (There are two more dimes than nickels)
3. \( x+y+z = 19 \)  \( \) (Mike has 19 coins)

**Step 1:** Use substitution to eliminate the \( y \) variable: Substitute \( y = x+2 \) into \( x+y+z = 19 \)

\[
\begin{align*}
x+y+z &= 19 \\
x+x+2 +z &= 19 & \text{Substitute } x+2 \text{ for } y \text{ in the equation, } x+y+z = 19 \\
2x+z+2 &= 19 & \text{Combine like terms} \\
2x+z &= 17 & \text{Equation #4}
\end{align*}
\]

**Step 2:** Eliminate the \( y \) variable by solving the system with the first and third equations.

\[
\begin{align*}
5x +10y +25z &= 235 \\
-10(x+y+z) &= 19(-10)
\end{align*}
\]

\[
\begin{align*}
-50x-10y-10z &= -190 \\
-5x +15z &= 45 & \text{Equation #5}
\end{align*}
\]

**Step 3:** Solve the system of equations for equations 4 and 5.

\[
\begin{align*}
-15(2x+z) &= 17(-15) \\
-5x +15z &= 45
\end{align*}
\]

\[
\begin{align*}
-30x -15z &= -255 \\
-5x +15z &= 45
\end{align*}
\]

\[
\begin{align*}
-35x &= -210 \\
x &= 6
\end{align*}
\]

**Step 4:** Substitute \( x = 6 \) into the equation \( y = x+2 \) in order to solve the equation for \( y \).

\[
\begin{align*}
Y &= 6+2 \\
y &= 8
\end{align*}
\]

**Step 5:** Substitute \( x = 6, y = 8 \) into the equation \( x +y+z = 19 \)

\[
\begin{align*}
6+8 +z &= 19 \\
14 +z &= 19 \\
14+14+z &= 19-14 \\
z &= 5
\end{align*}
\]

**Solution:** Mike has 6 nickels, 8 dimes, and 5 quarters.
JoAnne sold magazine subscriptions with three prices: $20, $17, and $15. She sold 2 fewer of the $20 subscriptions than of the $17 subscription and sold a total of 27 subscriptions. If her total sales amounted to $479, how many of each did she sell? (4 points)

**Define your variables and write the system of equations:**

Let \( x \) = number of $20 subscriptions
Let \( y \) = number of $17 dollar subscriptions
Let \( z \) = number of $15 dollar subscriptions

1. \( 20x + 17y + 15z = 479 \) Total sales amounted to $479.
2. \( x = y - 2 \) She sold 2 fewer $20 subscriptions than $17 dollar subscriptions
3. \( x + y + z = 27 \)

**Step 1:** Use substitution to eliminate the \( x \) variable: Substitute \( x = y - 2 \) into \( x + y + z = 27 \)

\[
x + y + z = 27
y - 2 + y + z = 27
2y + z - 2 = 27
2y + z - 2 + 2 = 27 + 2
2y + z = 29
\]

**Step 2:** Eliminate the \( x \) variable by solving the system with the 1\( ^{st} \) and 2\( ^{nd} \) equations.

\[
20x + 17y + 15z = 479
20(y-2) + 17y + 15z = 479
20y - 40 + 17y + 15z = 479
37y + 15z - 40 = 479
37y + 15z - 40 + 40 = 479 + 40
37y + 15z = 519
\]

**Step 3:** Solve the system of equations for equations 4 and 5.

\[
-15(2y + z) = 29(-15)
37y + 15z = 519
-30y - 15z = -435
\]

\[
37y + 15z = 519
7y = 84
Y = 12
\]

**Step 4:** Substitute \( y = 12 \) into the equation \( x = y - 2 \) in order to solve the equation for \( x \).

\[
X = y - 2
x = 12 - 2
x = 10
\]

**Step 5:** Substitute \( x = 10 \), \( y = 12 \) into the equation \( x + y + z = 27 \)

\[
10 + 12 + z = 27
22 + z = 27
Z = 5
\]

JoAnne sold 10 twenty dollar subscriptions, 12, seventeen dollar subscriptions, and 5 fifteen dollar subscriptions.
Part 1: Graphing System of Equations (Lesson 1)

1. Estimate the solution to the system of equations.

2. Graph the following system of equations and identify the solution.

   \[ Y = -3x + 8 \quad \& \quad 4x - 3y = -24 \]

   \[ y = -2x + 4 \quad \& \quad 5x - y = -4 \]
Part 2: Using the Substitution Method (Lesson 2)

Solve the following systems of equations using the substitution method.

1.  \( x = -3y \) &  \( 2x - 4y = 25 \)

2.  \( Y = -2x + 8 \) &  \( y = 3x - 2 \)

3. Solve the following system of equations and describe the graph.

   \( Y = -9x + 2 \) &  \( 27x + 3y = 6 \)

4. Solve the following system of equations.

   \( Y = \frac{1}{3}x - 4 \) &  \( y = \frac{1}{2}x + 4 \)

5. Mike has $320 and saves $5 per week. David has $200 and saves $9 per week. After how many weeks will they have the same amount of money? How much money will each have?

Part 3: Using Linear Combinations (Lesson 3)

1.  \( 2a - 3b = 17 \) &  \( a + 3b = 1 \)

2.  \( 2x - 2y = 1 \) &  \( 9x - y = -6 \) (Calculator needed)

3. Solve the system of equations and describe the graph.

   \( 3x + 4y = 2 \) &  \( 12y + 9x = 6 \)

4. The sum of two numbers is 112. The difference of the two numbers is 16. Write and solve a system of equations in order to identify the two numbers.
Part 4: YOU Choose the method. Solve the following system of equations using any method that you would like. (Lessons 1-3)

1. $x + 3y = 9$ & $2x - 2y = 34$

2. $x = 12 - 3y$ & $3x - 6y = 96$

Part 5: System of Equations Word Problems (Lesson 4)

1. The length of a flower bed is 3 times its width. The perimeter of the flower bed is 52 ft. Find the length of the flower bed.

2. Tickets to a play cost $4.50 for children and $6.50 for adults. A total of 806 tickets were sold and the total revenue was $4587.
   - Write a system of equations to represent this situation.
   - How many tickets of each were sold?

3. Renting a car from Company A costs $59 plus $0.10 a mile. Renting a car from Company B costs $39 plus $0.30 a mile.
   - For how many miles will the two companies cost the same amount?
   - If you plan on traveling for 150 miles, which company would you choose? Explain your answer.

Part 6 – Solving System in Three Variables (Lesson 5)

$\begin{cases} 
2x - 3y + z = -4 \\
x + 2y - 3z = -23 \\
3x + y - 2z = -23 
\end{cases}$

2. Mary has 26 coins (pennies, nickels and dimes) in her piggy bank with a total value of $1.40. There are 3 times as many nickels as pennies. How many of each coin does Mary have in her bank?
Part 1: Graphing System of Equations (Lesson 1)

1. Estimate the solution to the system of equations.

![Graph 1](Image 38x745 to 212x763)

**Solution (14,19)**

![Graph 2](Image 36x408 to 284x654)

**Solution (-3, -8)**
2. Graph the following system of equations and identify the solution.

\[ Y = -3x + 8 \text{ (blue)} \quad \& \quad 4x - 3y = -24 \text{ (red)} \]
\[ y = -2x + 4 \text{ (blue)} \quad \& \quad 5x - y = -4 \text{ (red)} \]

The first equation is written in slope intercept form:

\[ Y = -3x+8, \text{ so we identify the slope as } -3 \text{ and the y-intercept as 8.} \]

The second equation is written in standard form. You can either identify the x and y intercept or you can rewrite the equation in slope intercept form.

For this one, x and y intercept would be easier.

\[
\begin{align*}
4x - 3(0) &= -24 \\
4x &= -24 \\
4x/4 &= -24/4 \\
x &= -6
\end{align*}
\]

\[
\begin{align*}
4(0) - 3y &= -24 \\
-3y &= -24 \\
-3y/-3 &= -24/-3 \\
y &= 8
\end{align*}
\]

OR You could rewrite in slope intercept form:

\[
\begin{align*}
4x - 3y &= -4x - 24 \\
-3y &= -4x - 24 \\
-3y/-3 &= -4x/-3 - 24/-3 \\
Y &= 4/3x + 8
\end{align*}
\]

Now graph using 8 as the y-intercept and 4/3 as the slope.

The first equation is written in slope intercept form:

\[ Y = -2x + 4 \text{ so we identify the slope as } -2 \text{ and the y-intercept as 4.} \]

The second equation is written in standard form. For this equation, the intercepts will be fractions, so it would be much easier and more accurate to rewrite this equation in slope intercept form.

\[
\begin{align*}
5x - y &= -4 \\
5x - 5x - y &= -5x - 4 \\
-y &= -5x - 4 \\
-y/-1 &= -5x/-1 - 4/-1 \\
Y &= 5x + 4
\end{align*}
\]

Now graph using 5 as the slope and 4 as the y-intercept.
Part 2: Using the Substitution Method (Lesson 2)

Solve the following systems of equations using the substitution method.

1. \( x = -3y \) \& \( 2x - 4y = 25 \)

Since \( x = -3y \) is already solved for \( x \), we can go ahead and substitute \(-3y\) for \( x \) in the second equation.

\[
\begin{align*}
2x - 4y &= 25 \\
2(-3y) - 4y &= 25 & \text{Substitute } -3y \text{ for } x. \\
-6y - 4y &= 25 & \text{Simplify} \\
-10y &= 25 & \text{Combine like terms} \\
-10y/-10 &= 25/-10 & \text{Divide by } -10 \\
Y &= -2.5 \\
\end{align*}
\]

Now that we’ve found the value for \( y \), we can substitute \(-2.5\) into either equation and solve for \( x \).

The first equation would be easiest

\[
\begin{align*}
X &= -3y & \text{Original Equation} \\
X &= -3(-2.5) & \text{Substitute } -2.5 \text{ for } y \\
X &= 7.5 \\
\end{align*}
\]

The solution is: \((7.5, -2.5)\)

2. \( Y = -2x + 8 \) \& \( y = 3x - 2 \)

Let’s substitute \(-2x + 8\) for \( y \) into the second equation:

\[
\begin{align*}
Y &= 3x - 2 \\
-2x + 8 &= 3x - 2 & \text{Substitute } -2x + 8 \text{ for } y \\
-2x - 3x + 8 &= 3x - 3x - 2 & \text{Subtract } 3x \\
-5x + 8 &= -2 & \text{Simplify} \\
-5x + 8 - 8 &= -2 - 8 & \text{Subtract } 8 \\
-5x &= -10 & \text{Simplify} \\
-5x/-5 &= -10/-5 & \text{Divide by } -5 \\
X &= 2 \\
\end{align*}
\]

Now that we’ve found the value for \( x \), we can substitute \( 2 \) for \( x \) into either equation.

\[
\begin{align*}
Y &= 3x - 2 & \text{Original equation} \\
Y &= 3(2) - 2 & \text{Substitute } 2 \text{ for } x \\
Y &= 6 - 2 & \text{Simplify} \\
Y &= 4 & \text{Simplify} \\
\end{align*}
\]

The solution is: \((2,4)\)

3. Solve the following system of equations and describe the graph.

\[
\begin{align*}
Y &= -9x + 2 & \& \ 27x + 3y = 6 \\
\end{align*}
\]

I am going to choose the substitution method because the first equation is ready for substituting. It is already solved for \( y \). I can substitute \(-9x + 2\) for \( y \) into the second equation.

\[
\begin{align*}
27x + 3y &= 6 & \text{Original equation} \\
27x + 3(-9x + 2) &= 6 & \text{Substitute } -9x + 2 \text{ for } y \\
27x - 27x + 6 &= 6 & \text{Distribute the } 3 \\
0 + 6 &= 6 & \text{Simplify: } 27x - 27x = 0 \\
6 &= 6 & \text{Simplify} \\
\end{align*}
\]

Since I end up with \( 6 = 6 \), I know that the lines are either parallel or the same line. \( 6 = 6 \) is a true statement, therefore, the graph would show two identical lines lying on top of each other.
4. Solve the following system of equations.

\[ Y = \frac{1}{3}x - 4 \quad \& \quad y = \frac{1}{2}x + 4 \]

Since both equations are solved for \( y \), we can use the substitution method to solve the system.

\[ Y = \frac{1}{2}x + 4 \]

\[ \frac{1}{3}x - 4 = \frac{1}{2}x + 4 \]

Substitute \( \frac{1}{3}x - 4 \) for \( y \) into the second equation.

Now we have fractions to deal with. We need to get rid of the fractions by multiplying by the LCM of the denominators. This would be 6.

\[
6\left(\frac{1}{3}x - 4\right) = 6\left(\frac{1}{2}x + 4\right)
\]

Multiply by 6 on both sides.

\[
2x - 24 = 3x + 24
\]

Distribute the 6 through both parenthesis.

\[
2x - 3x - 24 = 3x - 3x + 24
\]

Subtract 3x from both sides.

\[
-x - 24 = 24
\]

Simplify: \( 2x - 3x = -x \)

\[
-x - 24 + 24 = 24 + 24
\]

Add 24 to both sides.

\[
-x = 48
\]

Simplify: \( 24 + 24 = 48 \)

\[
(-1) \cdot x = 48(-1)
\]

Multiply by -1 to make the variable positive.

\[ X = -48 \]

Now that we know \( x = -48 \), we can substitute -48 for \( x \) into either equation and solve for \( y \).

\[ Y = \frac{1}{2}x + 4 \]

\[ Y = \frac{1}{2}(-48) + 4 \]

\[ Y = -24 + 4 \]

\[ Y = -20 \]

The solution to this system of equations is: \((-48, -20)\)
5. Mike has $320 and saves $5 per week. David has $200 and saves $9 per week. After how many weeks will they have the same amount of money? How much money will each have?

We must first write two equations that describe this situation. One equation for Mike and one equation for David.

Mike: \( y = 5x + 320 \)  
David: \( y = 9x + 200 \)  
Let \( x \) = the number of weeks  
Let \( y \) = total amount

We can solve the system to find out how many weeks it will take them to have the same amount of money.

\[
\begin{align*}
5x + 320 &= 9x + 200 \\
5x - 9x + 320 &= 9x - 5x + 200 \\
-4x + 320 &= 4x + 200 \\
-320 + 200 &= 4x - 200 \\
-120 &= 4x \\
120/4 &= 4x/4 \\
30 &= x
\end{align*}
\]

After 30 weeks they will have the same amount of money.

To figure out how much money they will have, we will solve for \( y \).

\[
\begin{align*}
Y &= 5x+320 \\
Y &= 5(30) + 320 \\
Y &= 470
\end{align*}
\]

They will both have $470 dollars after 30 weeks.

Part 3: Using Linear Combinations (Lesson 3)

1. \( 2a - 3b = 17 \ & \ a + 3b = 1 \)

Since both equations are written in standard form, we can use linear combinations. Let’s set them up:

\[
\begin{align*}
2a - 3b &= 17 \\
a + 3b &= 1 \\
\end{align*}
\]

\[
\begin{align*}
3a &= 18 \\
\text{Add remaining terms} \\
3a/3 &= 18/3 = 6 \\
a &= 6 \\
\text{Simplify:} \\
\text{Now solve for a.} \\
3a/3 &= 18/3 = 6 \\
a &= 6 \\
\text{Simplify:} \\
\text{Substitute 6 for a} \\
6 + 3b &= 1 \text{ Substitute 6 for a} \\
6 - 6 + 3b &= 1 - 6 \text{ Subtract 6 from both sides} \\
3b &= -5 \text{ Simplify} \\
3b/3 &= -5/3 \text{ Divide by 3 on both sides} \\
b &= -5/3
\end{align*}
\]

The solution is: \((6, -5/3)\)

2. \( 2x - 2y = 1 \ & \ 9x - y = -6 \) \text{(Calculator needed)}

Since both equations are written in standard form, we can use linear combinations. Let’s set them up:

\[
\begin{align*}
2x - 2y &= 1 \\
9x - y &= -6 \\
\end{align*}
\]

\[
\begin{align*}
2x - 2y &= 1 \text{ No terms cancel, so I will} \\
9x - y &= -6 \text{ Multiply the 2\textsuperscript{nd} equation by -2} \\
2x - 2y &= 1 \\
9x - y &= -6 \text{ Multiply the 2\textsuperscript{nd} equation by -2} \\
-18x + 2y &= 12
\end{align*}
\]

\[
\begin{align*}
2x - 2y &= 1 \text{ Now -2y and 2y cancel} \\
-18x + 2y &= 12 \\
-16x &= 13 \\
-16x/-16 &= 13/-16 \\
X &= -8.125
\end{align*}
\]

Now substitute \(-8.125\) for \(x\) into one of the equations and solve for \(y\).

\[
\begin{align*}
9(-8.125) - y &= -6 \text{ Substitute} \\
-73.125 - y &= -6 \\
-73.125 + 7.3125 - y &= -6 + 7.3125 \text{ Add 7.3125} \\
(-1)y &= (-1)3.125 \text{ Multiply by -1} \\
Y &= -1.3125 \text{ solution:} \ (-8.125, -1.3125)
\end{align*}
\]
3. Solve the system of equations and describe the graph.

\[ 3x + 4y = 2 \quad \text{and} \quad 12y + 9x = 6 \]

Since both equations are written in standard form, I will use the linear combination method to solve the system.

Make sure that you line the variables up correctly. The second equation is written with the variables out of order. We must multiply in order to create opposite terms. I will multiply the first equation by -3.

\[ -3(3x + 4y = 2) \rightarrow -9x -12y = -6 \quad \text{Now we have opposite terms} \]
\[ 9x +12y = 6 \quad \text{All terms cancel. Since 0=0 is a true statement, this system of equation is the same line.} \]

Since the equations are the same line, on a graph the lines would lay one on top of each other.

4. The sum of two numbers is 112. The difference of the two numbers is 16. Write and solve a system of equations in order to identify the two numbers.

First we need to define variables for the two numbers: Let \( x \) = one number \quad \text{Let } y = \text{the other number} \]

The sum of two numbers is 112. A sum is the answer to an addition problem, so…
\[ x + y = 112 \]

The difference of the two numbers is 16. A difference is the answer to a subtraction problem, so…
\[ x - y = 16 \]

Now we have a system of equations: \( x + y = 112 \quad \text{and} \quad x - y = 16 \)

We can use linear combinations or substitution in order to solve. Since I have opposite \( y \) terms, I will use linear combinations.

\[ \begin{align*}
3x + 4y &= 2 \\
9x + 12y &= 6 \\
2x &= 128 \\
x &= 64 \\
\end{align*} \]

Now let’s take \( x = 64 \) and substitute it for \( x \) into one of the equations and solve for \( y \).

\[ \begin{align*}
x + y &= 112 \\
64 + y &= 112 \\
64 - 64 + y &= 112 - 64 \\
Y &= 48 \\
\end{align*} \]

The two numbers are: 64 and 48.
Part 4: YOU Choose the method. Solve the following system of equations using any method that you would like. (Lessons 1-3)

1. \( x + 3y = 9 \) & \( 2x - 2y = 34 \)

You could use substitution by solving the first equation for \( x \):

\[ x + 3y - 3y = 9 - 3y \]
\[ x = -3y + 9 \]

Now substitute \( 3y + 9 \) for \( x \) into the other equation.

\[ 2x - 2y = 34 \]
\[ 2(-3y + 9) - 2y = 34 \]
\[ -6y + 18 - 2y = 34 \]
\[ -8y = 16 \]
\[ -8y/-8 = 16/-8 \]
\[ y = -2 \]

Now substitute \(-2\) for \( y \) into either equation and solve for \( x \).

\[ x + 3(-2) = 9 \]
\[ x - 6 = 9 \]
\[ x - 6 + 6 = 9 + 6 \]
\[ x = 15 \]

The solution is \((15, -2)\)

Or, you could solve using linear combinations:

\[ -2(x + 3y) = 9(-2) \rightarrow -2x - 6y = -18 \]
\[ 2x - 2y = 34 \]
\[ -8y = 16 \]
\[ -8y/-8 = 16/-8 \]
\[ y = -2 \]

Now substitute \(-2\) for \( y \) into either equation (as we did above) and you will find that \( x = 15 \).

2. \( X = 12 - 3y \) & \( 3x - 6y = 96 \)

Since this equation is already solved for \( x \), it would only make sense to use the substitution method.

Substitute \( 12 - 3y \) for \( x \) into the second equation:

\[ 3x - 6y = 96 \]
\[ 3(12 - 3y) - 6y = 96 \]
\[ 36 - 9y - 6y = 96 \]
\[ -15y = 60 \]
\[ -15y/-15 = 60/-15 \]
\[ y = -4 \]

Now that we know \( y = -4 \), we can substitute \(-4\) for \( y \) into either equation and solve for \( x \). The easiest equation to use would be equation #1 since it is already solved for \( x \).

\[ X = 12 - 3y \]
\[ X = 12 - 3(-4) \]
\[ X = 24 \]

The solution is \((24, -4)\)
Part 5: System of Equations Word Problems (Lesson 4)

1. The length of a flower bed is 3 times its width. The perimeter of the flower bed is 52 ft. Find the length of the flower bed.

   Let \( w \) = the width of the flower bed \hspace{1cm} \text{Perimeter is 52}

   \( 3w \) is the length: \( \text{(the length is 3 times its width)} \)

   \[
   \begin{align*}
   2l + 2w &= P \\
   2(3w) + 2w &= 52 \\
   6w + 2w &= 52 \\
   8w &= 52 \\
   8w/8 &= 52/8 \\
   W &= 6.5 \text{ ft} \hspace{1cm} \text{The width is 6.5 ft.}
   \end{align*}
   \]

   Now that we know the width, we can find the length by substituting 6.5 for \( w \) into the perimeter formula.

   \[
   \begin{align*}
   2l + 2(6.5) &= 52 \\
   2l + 13 &= 52 \\
   2l + 13 - 13 &= 52 - 13 \\
   2l &= 39 \\
   2l/2 &= 39/2 \\
   L &= 19.5 \text{ ft} \hspace{1cm} \text{The length is 19.5 feet}
   \end{align*}
   \]

2. Tickets to a play cost $4.50 for children and $6.50 for adults. A total of 806 tickets were sold and the total revenue was $4587.

   - Write a system of equations to represent this situation.

   Let \( x \) = the number of children tickets \hspace{1cm} \text{Let} \ y = \text{the number of adult tickets}

   Ticket Price: \( 4.5x + 6.5y = 4587 \)
   Number of Tickets: \( x + y = 806 \)

   The system of equations that represents this situation is: \( 4.5x + 6.5y = 4587 \) \hspace{1cm} & \hspace{1cm} x + y = 806
• How many tickets of each were sold?

In order to find how many tickets of each were sold, we must solve the system of equations. I am going to first solve the second equation for x and use the substitution method.

\[ x + y - y = 806 - y \]
\[ x = 806 - y \]

Now substitute 806 - y for x into the first equation.

\[ 4.5(806 - y) + 6.5y = 4587 \]
\[ 3627 - 4.5y + 6.5y = 4587 \]
\[ 3627 + 2y = 4587 \]
\[ 2y = 960 \]
\[ y = 480 \]

480 adult tickets were sold.

Now substitute 480 for y into either equation and solve for x.

\[ x + y = 806 \]
\[ x + 480 = 806 \]
\[ x = 326 \]

326 children’s tickets and 480 adult tickets were sold.

3. Renting a car from Company A costs $59 plus $0.10 a mile. Renting a car from Company B costs $39 plus $0.30 a mile.

• For how many miles will the two companies cost the same amount?

Let’s start by writing a system of equations. An equation for Company A and an equation for Company B

Let \( x \) = the number of miles \hspace{1cm} \text{Let } y = \text{total cost}

Company A: \( y = .10x + 59 \)
Company B: \( y = .30x + 39 \)

Now we’ll solve using the substitution method.

\[ .10x + 59 = .30x + 39 \]
\[ .10x -.30x + 59 = .30x -.30x + 39 \]
\[ -.20x + 59 = 39 \]
\[ -.20x + 59 - 59 = 39 - 59 \]
\[ -.20x = -20 \]
\[ -.20x / -.20 = -20 / -.2 \]
\[ x = 100 \]

Since we only need to answer for how many miles will the two companies charge the same amount, we can stop here. The two companies will charge the same amount for 100 miles.
If you plan on traveling for 150 miles, which company would you choose? Explain your answer.

If you plan on traveling for 150 miles, you would choose **company A**. Company B is cheaper up until 100 miles and then company A becomes cheaper for any amount of miles over 100 miles.

Justify:

**Company A**: \(0.1(150) + 59 = 74\)

**Company B**: \(0.3(150) + 39 = 84\)
Part 6 – Solving System in Three Variables (Lesson 5)

1. \[2x - 3y + z = -4\]
2. \[x + 2y - 3z = -23\]
3. \[3x + y - 2z = -23\]

Step 1: Equations 1 and 2 – Get rid of x variable.

\[2x - 3y + z = -4\]
\[-2(x + 2y - 3z) = -23(-2)\]
\[-7y + 7z = 42\]

Step 2: Equations 2 and 3 – Get rid of x variable.

\[-3(x + 2y - 3z) = -23(-3)\]
\[3x + y - 2z = -23\]
\[-5y + 7z = 46\]

Step 3: Solve the system using Equations 4 and 5.

\[-7y + 7z = 42\]
\[-5y + 7z = 46\]
\[2y = 4\]
\[y = 2\]

Step 4: Substitute 2 for y into the equation, \(-7y + 7z = 42\)

\[-7y + 7z = 42\]
\[-7(2) + 7z = 42\]
\[-14 + 7z = 42\]
\[-14z + 7z = 42 + 14\]
\[7z = 56\]
\[z = 8\]

Step 5: \(y = 2\) and \(z = 8\) into one of the original equations

\[x + 2y - 3z = -23\]
\[x + 2(2) - 3(8) = -23\]
\[x + 4 - 24 = -23\]
\[x - 20 = -23\]
\[x - 20 + 20 = -23 + 20\]
\[x = -3\]

Solution: \((2, -3, 8)\)
2. Mary has 26 coins (pennies, nickels and dimes) in her piggy bank with a total value of $1.40. There are 3 times as many nickels as pennies. How many of each coin does Mary have in her bank?

Let \( p \) = the number of pennies  
Let \( n \) = the number of nickels  
Let \( d \) = the number of dimes

- \( p + 5n + 10d = 140 \)  
  total amount $140
- \( n = 3p \)  
  3 times as many nickels as pennies
- \( p + n + d = 26 \)  
  she has 26 coins.

\[
\begin{align*}
\text{Step 1: Substitute } n &= 3p \text{ into the equation } p+n+d = 26 \\
p+3p +d &= 26 \\
4p +d &= 26 \\
\text{Combine like terms}
\end{align*}
\]

\[
\begin{align*}
\text{Step 2: Substitute } n=3p \text{ into equation #1} \\
P+5n+10d &= 140 \\
P+5(3p) +10d &= 140 \\
P+15p +10d &= 140 \\
16p +10d &= 140 \\
\text{Combine like terms}
\end{align*}
\]

\[
\begin{align*}
\text{Step 3: Solve equations 4 and 5 for } d. \\
-10(4p +d) &= 26(-10) \\
-40p -10d &= -260 \\
16p +10d &= 140 \\
16p +10d &= 140 \\
24p &= 120 \\
P &= 5
\end{align*}
\]

\[
\begin{align*}
\text{Step 4: Substitute } p = 5 \text{ into the equation } n = 3p \\
N=3p \\
N = 3(5) \\
N = 15
\end{align*}
\]

\[
\begin{align*}
\text{Step 5: Substitute } p = 5 \text{ and } n = 15 \text{ into one of the equations.} \\
P+n+d &= 26 \\
5+15 +d &= 26 \\
20+d &= 26 \\
d &= 6 \\
\text{He has 5 pennies, 15 nickels and 6 dimes.}
\end{align*}
\]
Systems of Equations Chapter Test

1. Estimate the solution to the system of equations. (1 Point)

   ![Graph showing two lines intersecting at a point]

   A. (-1, 6)  
   B. (6, -1)  
   C. (6,1)   
   D. (-1,6)

2. Which answer best describes the solution to the system of equations? (1 point)

   \[ \begin{align*}
   x &= 5 - 9y \\
   4x + 9y &= -7
   \end{align*} \]

   A. (1, -4)  
   B. (-4,1)  
   C. (-1,4)  
   D. (-9,5)

3. Which of the following systems of equations has no solution? (1 point)

   A. \[ \begin{align*}
   2x + y &= 4 \\
   4x + 2y &= -12
   \end{align*} \]
   
   B. \[ \begin{align*}
   2x + y &= 4 \\
   2x + 3y &= 12
   \end{align*} \]
   
   C. \[ \begin{align*}
   2x + y &= 4 \\
   6x + 2y &= 8
   \end{align*} \]
   
   D. None of the above.
4. Which ordered pair is the solution to the system of equations? (1 point)
\[
\begin{align*}
6x - y &= -4 \\
2x + 2y &= 15
\end{align*}
\]
\begin{align*}
A. (2,7) & \quad C. (1/2, 7) \\
B. (7,2) & \quad D. (2, 16)
\end{align*}

5. You are selling tickets to the local carnival as a fundraiser. Each adult ticket cost $7 and each child ticket cost $5. You sell 120 tickets and collect $680. Which system of equations represents this situation? (1 Point)
\begin{align*}
A. 7x + 5y &= 120 \\
& \quad x + y = 680 \\
C. y &= 5x + 120 \\
& \quad y = 7x + 680 \\
B. y &= 7x + 120 \\
& \quad y = 5x + 680 \\
D. 7x + 5y &= 680 \\
& \quad x + y = 120
\end{align*}

6. Solve the system of equations. (1 Point)
\[
\begin{align*}
y &= -x + 5 \\
x - 4y &= 10
\end{align*}
\]
\begin{align*}
A. (-1,6) & \quad C. (-6,1) \\
B. (6, -1) & \quad D. (No Solution)
\end{align*}
7. Which system of equations has an infinite number of solutions?  (1 Point)

A. 3x – y = -4  
   3x – 2y = -8  
B. 3x – y = -4  
   y = -3x -4  
C. 3x – y = -4  
   y = 3x - 4  
D. 3x – y = -4  
   9x – 3y = -12

8. The second of two numbers is 8 more than the first. The sum of the numbers is 52. Find the numbers. (2 Points)

9. Jerry wants to rent an aerator to prepare this lawn for the spring and summer. One company charges $5 an hour, plus $25 for delivery and pick up. The second company charges $8 an hour, plus $10 for delivery and pick up. For how many hours will the two companies charge the same amount? (1 Point)

A. 5 hours  
B. 4 hours  
C. 6 hours  
D. None of the above.

10. Which statement best describes the graph of the system of equations? (1 Point)

\[ y = -8x + 3 \]
\[ 16x + 2y = -12 \]

A. Two perpendicular lines  
B. Two parallel lines  
C. Two lines intersecting at (6,-12)  
D. Two identical lines
11. Mulch It charges $16 per yard of mulch plus a $20 delivery fee. Weber’s Mulch charges $13 per yard of mulch plus a $29 delivery fee. After how many yards does Mulch It become more expensive? (1 Point)

A. 3 yards  
B. 4 yards  
C. 2 yards  
D. None of the above.

12. The following system of equations represents three different planes in space. Find the point of intersection of these planes. (1 Point)

\[
\begin{align*}
2x + 3y + z &= 21 \\
x - 2y + 2z &= -23 \\
3x + y - 3z &= -8
\end{align*}
\]

A. (10, -5, 1)  
B. (5, 10, -1)  
C. (-5, 10, 1)  
D. (1, -5, 10)

13. Graph the following system of equations and find identify the solution. (2 Points)

\[
\begin{align*}
y &= 4x - 6 \\
2x + y &= 6
\end{align*}
\]
14. Solve the following system of equations. (2 points)
\[
\begin{align*}
2x - 3y + z &= 30 \\
-2x + y - 3z &= -14 \\
x - y + 2z &= 11
\end{align*}
\]

15. The length of a flower bed is 1.5 times its width. The perimeter of the flower bed is 40 ft. Find the length of the flower bed. (2 Points)

16. The number of calories in a piece of cake is 30 more than the number of calories in a scoop of ice cream. The cake and ice cream together have 610 calories. How many calories are in each? (2 Points)

17. The girl scouts are selling boxes of cookies. They pay $20 for a contract with the company and $6 a box for the cookies. They sell the cookies for $8 a box. (3 Points)

- Write a system of equations that represents this situation.
- How many boxes of cookies must the Girl Scouts sell in order to break even? Justify your answer.
18. During a conference lunches were ordered by 2 groups of people. See the table below. (4 Points)

<table>
<thead>
<tr>
<th></th>
<th>Number of Chicken Lunches</th>
<th>Number of Pasta Lunches</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>14</td>
<td>$176.50</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>19</td>
<td>$172.75</td>
</tr>
</tbody>
</table>

- Write a system of equations that represents this situation.
- What is the price of a chicken lunch?
- What is the price of a pasta lunch?
- Explain how you determined your answers.
Systems of Equations Chapter Test – Answer Key

1. Estimate the solution to the system of equations. (1 Point)

The solution is the point where the 2 lines intersect:

(6, -1)

A. (-1, 6)       C. (6,1)
B. (6, -1)       D. (-1,6)

2. Which answer best describes the solution to the system of equations? (1 point)

\[
x = 5 - 9y \\
4x + 9y = -7
\]

A. (1, -4)       C. (-1,4)
B. (-4,1)       D. (-9,5)

Substitution is the most efficient method to use to solve this problem.

<table>
<thead>
<tr>
<th>Step 2: Solve for x:</th>
<th>4(5-9y) + 9y = -7</th>
<th>Substitute 5 – 9y for x into the 2\textsuperscript{nd} equation.</th>
<th>Y = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20 – 36y +9y = -7</td>
<td>Distribute the 4 throughout the parenthesis.</td>
<td>X = 5 -9y</td>
</tr>
<tr>
<td></td>
<td>20 – 27y = -7</td>
<td>Combine like terms (-36y + 9y = -27y)</td>
<td>x = 5 – 9(1)</td>
</tr>
<tr>
<td></td>
<td>20 -20 – 27y = -7 -20</td>
<td>Subtract 20 from both sides.</td>
<td>x = -4</td>
</tr>
<tr>
<td></td>
<td>-27y = -27</td>
<td>Simplify: -7 -20 = -27</td>
<td>The solution is: (-4,1)</td>
</tr>
<tr>
<td></td>
<td>-27</td>
<td>Divide both sides by -27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>y = 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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3. Which of the following systems of equations has no solution? (1 point)

A. \(2x + y = 4\)
   \(4x + 2y = -12\)

B. \(2x + y = 4\)
   \(2x + 3y = 12\)

C. \(2x + y = 4\)
   \(6x + 2y = 8\)

D. None of the above.

If a system of equations has no solution, then you know that you have parallel lines. Parallel lines have the same slope, but different y-intercepts. You can solve this 2 ways: The first way is to rewrite each equation in slope intercept form and choose the one with the same slope and different y-intercepts. The second way is to solve each using linear combinations (or substitution) and choose the one that has a result that doesn’t make sense. I’ll show both ways.

**Method 1: Rewrite in Slope Intercept Form**

A. \(2x + y = 4\)
   \(4x + 2y = -12\)

\[2x - 2x + y = 4 - 2x\]
\[4x - 4x + 2y = -12 - 4x\]

\[y = -2x + 4\]
\[2y = -4x - 12\]
\[\frac{2y}{2} = \frac{-4x}{2} - \frac{12}{2}\]

\[Y = -2x - 6\]

We lucked out! Letter A shows 2 equations that have the same slope (-2x) and different y-intercepts! Therefore, these two lines are parallel and there is no solution! The correct answer is letter A.

**If letter A did not have 2 equations that had the same slope and different y-intercepts, then we would have to repeat that process for letters b, and c until we found the correct answer!**

Now I’ll show you what happens if you use linear combinations!

\[-2(2x + y = 4)\]
\[4x + 2y = -12\]

\[-4x - 2y = -8\]
\[4x + 2y = -12\]

\[0 = -20\]

Multiply by -2 to create opposite x terms
Keep this equation the same.

\[0 = -20\] is NOT a true statement; therefore, there is no solution!
4. Which ordered pair is the solution to the system of equations? (1 point)

\[
\begin{align*}
6x - y &= -4 \\
2x + 2y &= 15
\end{align*}
\]

A. (2, 7)       C. \((1/2, 7)\)

B. (7, 2)       D. (2, 16)

If the ordered pair is a solution to the system of equations, then when you substitute \(x\) and \(y\) into each equation, the result will be a true statement. Therefore, there are 3 ways that you could solve this problem.

1. You could use linear combinations since both are written in standard form. (I think this is the easiest method)

2. You could use substitution since the first equation could easily be rewritten as \(y = \ldots\) (This requires an extra step, but can still be used!)

3. Since it is multiple choice, you could substitute the ordered pairs into each equation and see which answer makes sense. (Some see this as a shortcut, but I actually think this could take you longer, especially if the correct answer is the last one you try!)

The choice is yours, depending on which method you favor. I personally would choose the combination method. I will demonstrate the combination method and show how you can use method 3 to check your answer!

**Method 1: Linear Combinations**

\[
\begin{align*}
2(6x - y &= -4) &\quad 12x - 2y = -8 \\
2x + 2y &= 15 &\quad \text{Leave this equation as is.}
\end{align*}
\]

\[
\begin{align*}
14x &= 7 &\quad \text{Add like terms.}
\end{align*}
\]

\[
\begin{align*}
14 &\quad 14
\end{align*}
\]

\[
\begin{align*}
x &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
6x - y &= -4 \\
6(1/2) - y &= -4 \\
3 - y &= -4 \\
3 - 3 - y &= -4 - 3 \\
-y &= -7 \\
-1 &= -1
\end{align*}
\]

\[
\begin{align*}
Y &= 7
\end{align*}
\]

The ordered pair is \((1/2, 7)\) This is the solution.

**Method 3: Substituting the answers into the original equations:**

\[
\begin{align*}
6x - y &= -4 &\quad 2x + 2y &= 15 \\
6(1/2) - 7 &= -4 &\quad 2(1/2) + 2(7) &= 15 \\
3 - 7 &= -4 &\quad 1 + 14 &= 15
\end{align*}
\]

When you substitute \(\frac{1}{2}\) for \(x\) and 7 for \(y\) into each equation, the end result is a true statement.
5. You are selling tickets to the local carnival as a fundraiser. Each adult ticket cost $7 and each child ticket cost $5. You sell 120 tickets and collect $680. Which system of equations represents this situation? (1 Point)

A. \(7x + 5y = 120\) \(x + y = 680\)  
B. \(y = 7x + 120\) \(y = 5x + 680\)  
C. \(y = 5x + 120\) \(y = 7x + 680\)  
D. \(7x + 5y = 680\) \(x + y = 120\)

In this problem, you are given information about the number of tickets sold and the cost of the tickets. In order to write a system of equations, we must write 2 equations, 1 for the number of tickets and one for the cost of the tickets.

Let \(x\) = the number of adult tickets sold  
Let \(y\) = the number of children’s tickets sold

\[x + y = 120\]  
This equation represents the number of tickets sold. (\# of adult tickets + \# of child tickets = 120)

\[7x + 5y = 680\]  
This equation represents the cost of the tickets. (\$7 • \# of adult tickets + \$5 • \# of child tickets = \$680.)

6. Solve the system of equations. (1 Point)

\[y = -x + 5\] \[x - 4y = 10\]

A. (-1, 6)  
B. (6, -1)  
C. (-6, 1)  
D. No Solution

Since this system of equation is set up so that one of the equations is in slope intercept form (\(y = -x + 5\)) and one is written in standard form, the best method to use is substitution. You can substitute the value for \(y\) (-\(x + 5\)) into the second equation and solve for \(x\). You can also rewrite the first equation in standard form and use linear combinations, but this would require extra steps, and may not be worth the effort.

\[y = -x + 5\]
\[x - 4y = 10\]

Substitute -\(x + 5\) for \(y\) into the second equation.  
\[x = 6\]
\[y = -x + 5\]

Distribute -4 throughout the parenthesis.  
Combine like terms \((x + 4x = 5x)\)  
Add 20 to both sides  
Divide by 5  
The ordered pair is (6, -1)
7. Which system of equations has an infinite number of solutions? (1 Point)

A. \(3x - y = -4\)
   \(3x - 2y = -8\)

B. \(3x - y = -4\)
   \(y = -3x - 4\)

C. \(3x - y = -4\)
   \(y = 3x - 4\)

D. \(9x - 3y = -12\)

If a system of equations has an infinite number of solutions, then the two lines are equal – they are the same line! Therefore, the equations should be exactly the same! The easiest way to figure out the solution to this problem is to write both equations in the same form and see which one has two equations that are exactly the same.

A. Both equations are already written in standard form. They are not the same; however let’s check to see if one them can be simplified. Can you divide every term in the equation by a number to simplify it without creating fractions? No, it’s not possible! Therefore, letter A is not the solution.

B. One is written in standard form and one in slope intercept form. Let’s rewrite the first one in slope intercept form and compare.

\[
\begin{align*}
3x - y &= -4 \\
3x + 4 &= y \quad \text{OR} \quad y = 3x + 4 \\
\end{align*}
\]

Compare to: \(y = -3x - 4\)  They are not the same equation!

C. Same scenario as letter B. Let’s rewrite the first equation in slope intercept form and compare.

\[
\begin{align*}
3x - y &= -4 \\
3x - 3x - y &= -4 - 3x \\
-y &= -3x - 4 \\
-1 &= -1 \\
\end{align*}
\]

\[Y = 3x + 4\]  Compare to: \(y = 3x - 4\)  They are not the same equation!

D. The answer MUST be D, but we must verify!

The equations are both written in standard form, but they don’t look like the same equation! Let’s see if one of them can be simplified. Can you take one of the equations, and divide all terms by a number and be left with an equation that doesn’t contain fractions? Yes! The second equation: The coefficients and constants are all divisible by 3!

\[
\begin{align*}
9x - 3y &= -12 \\
3 &= 3 \\
\end{align*}
\]

\[3x - y = -4\]  Compare to: \(3x - y = -4\)  They ARE the SAME equation! This is the correct answer. Since both equations are exactly the same, then the two lines are equal and therefore, there are an infinite number of solutions!
8. The second of two numbers is 8 more than the first. The sum of the numbers is 52. Find the numbers. (2 Points)

In order to find the numbers, we can write a system of equations and solve!

I know the sum of the two numbers is 52. A sum is the answer to an addition problem. Since I don’t know either number, they will be my variables.

Let \( x \) = the first number \hspace{1cm} \text{Let} \ y = \text{the second number}

\[
\begin{align*}
x + y &= 52 \quad \text{The sum of the two numbers is 52} \\
y &= x + 8 \quad \text{The 2\textsuperscript{nd} number is 8 more than the first.}
\end{align*}
\]

Now that I have two equations, I can solve to find \( x \) and \( y \), which are my two numbers.

Since one equation is in slope intercept form \((y = x + 8)\) and the other is in standard form, I am going to use substitution to solve this system of equations.

\[
\begin{align*}
y &= x + 8 \\
x + y &= 52 \\
x + (x + 8) &= 52 \\
2x + 8 &= 52 \\
2x + 8 - 8 &= 52 - 8 \\
2x &= 44 \\
\frac{2x}{2} &= \frac{44}{2} \\
x &= 22
\end{align*}
\]

\[
\begin{align*}
y &= x + 8 \\
y &= 22 + 8 \\
y &= 30
\end{align*}
\]

The two numbers are 22 and 30!

\[
\begin{align*}
\text{Justify:} & \quad 22 + 30 = 52 \\
\text{Justify:} & \quad 30 = 22 + 8
\end{align*}
\]
I am given information about two different companies. For each company, I am given the rate ($ per hour) and a constant (delivery fee). Therefore, I have the slope and y-intercept and I can write my equations in slope intercept form. Since the question asks for the number of hours when they charge the same amount, I will need to solve this system of equations.

Company 1: \( y = 5x + 25 \)

Company 2: \( y = 8x + 10 \)

Since I have two equations that are written in slope intercept form, I am going to use substitution to solve!

\[
\begin{align*}
Y &= 5x + 25 \\
5x + 25 &= 8x + 10 \\
5x - 5x + 25 &= 8x + 10 - 5x \\
25 &= 3x + 10 \\
25 - 10 &= 3x - 10 + 10 \\
15 &= 3x \\
3x &= 15 \\
x &= 5
\end{align*}
\]

The number of hours that the two companies will charge the same amount is 5!

Check your work:

\[
\begin{align*}
Y &= 5x + 25 \\
y &= 8x + 10 \\
y &= 5(5) + 25 \\
y &= 8(5) + 10 \\
y &= 50 \\
y &= 50
\end{align*}
\]

Both companies charge $50 for 5 hours.
10. Which statement best describes the graph of the system of equations? (1 Point)

   \[ y = -8x + 3 \]
   \[ 16x + 2y = -12 \]

   A. Two perpendicular lines  
   B. Two parallel lines  
   C. Two lines intersecting at (6,-12)  
   D. Two identical lines

This may seem tricky since you don’t have a graph! We can tell a lot about the graph by solving the system of equations! Let’s solve this system using the substitution method since one equation is written in slope intercept form and one is in standard form.

\[ y = -8x + 3 \]
\[ 16x + 2y = -12 \]

\[ 16x + 2(-8x+3) = -12 \]  
Substitute -8x +3 for y into the second equation.

\[ 16x -16x +6 = -12 \]  
Distribute the 2 throughout the parenthesis

\[ 6 = -12 \]  
The x terms cancelled out and the expression 6 = -12 is NOT a true statement. This means that there is NO Solution to this system. If there is no solution to a system of equations, then the lines are parallel.
11. Mulch It charges $16 per yard of mulch plus a $20 delivery fee. Weber’s Mulch charges $13 per yard of mulch plus a $29 delivery fee. After how many yards does Mulch It become more expensive? (1 Point)

A. 3 yards  C. 2 yards  
B. 4 yards  D. None of the above.

We are given information about Mulch It and Weber’s Mulch. Let’s write an equation for each company. For each company we are given the rate ($ per yard) and a constant (delivery fee). Therefore, we can write both equations in slope intercept form.

**Mulch It:** \( y = 16x + 20 \)

**Weber’s** \( y = 13x + 29 \)

They key word in this problem, is “After” how many yards will Mulch It become more expensive. What typically happens with Systems of Equations is one company will be more expensive initially, then there will be the point where they cost the same amount, after that point the other company will become more expensive.

So, we need to find the point where the 2 companies charge the same, because “After” that point, Mulch It will be more expensive. So, let’s solve the system of equations using substitution.

\[
\begin{align*}
16x + 20 & = 13x + 29 \\
16x - 13x + 20 & = 13x - 13x + 29 \\
3x + 20 & = 29 \\
3x + 20 - 20 & = 29 - 20 \\
3x & = 9 \\
\frac{3x}{3} & = \frac{9}{3} \\
x & = 3
\end{align*}
\]

We know they charge the same amount for 3 yards, so **After 3 yards**, Mulch should be more expensive. Let’s check and see by substituting 4 for x into both equations.

**Mulch it:** \( y = 16x + 20 \)  \( Y = 16(4) + 20 \) \( Y = 84 \)

**Weber’s** \( y = 13x + 29 \)  \( y = 13(4) + 29 \)  \( y = 81 \)

**More expensive**  **So, after 3 yards, Mulch It will be more expensive.**
12. The following system of equations represents three different planes in space. Find the point of intersection of these planes.

\[
\begin{align*}
2x + 3y + z &= 21 \\
x - 2y + 2z &= -23 \\
3x + y - 3z &= -8
\end{align*}
\]

A. (10, -5, 1)  B. (5, 10, -1)  C. (-5, 10, 1)  D. (1, -5, 10)

It would be best to start solving this system, rather than trying to substitute each answer.

**Equations 1 & 2**

\[
\begin{align*}
2x + 3y + z &= 21 \\
-2(x-2y +2z) &= -23(-2)
\end{align*}
\]

\[
\begin{align*}
-2x + 4y -4z &= 46 \\
7y - 3z &= 67
\end{align*}
\]

**Equations 2 & 3**

\[
\begin{align*}
-3(x-2y +2z) &= -23(-3) \\
3x + y - 3z &= -8
\end{align*}
\]

\[
\begin{align*}
-3x +6y -6z &= 69 \\
7y - 9z &= 61
\end{align*}
\]

**Equations 4&5**

\[
\begin{align*}
-1(7y - 3z) &= 67(-1) \\
7y - 9z &= 61
\end{align*}
\]

\[
\begin{align*}
-7y +3z &= -67 \\
7y -9z &= 61
\end{align*}
\]

\[
\begin{align*}
-6z &= -6 \\
z &= 1
\end{align*}
\]

Since \( z = 1 \), we can eliminate B and D.

Let's substitute and try A.

\[
\begin{align*}
2(10) +3(-5)+1 &= 21 \\
6 &\neq 21
\end{align*}
\]

Let's try C:

\[
\begin{align*}
2(-5) +3(10) +1 &= 21 \\
-5 -2(10) +2(1) &= -23 \\
3(-5) +10 - 3(1) &= -8
\end{align*}
\]

Answer choice C is correct.

13. Graph the following system of equations and find the identify the solution. (2 Points)

In order to graph the two equations, we need both equations to be written in slope intercept form. The first equation is already written in slope intercept form. The second equation needs to be rewritten.

\[
y = 4x - 6
\]

\[
2x + y = 6
\]

\[
2x - 2x + y = 6 - 2x
\]

\[
y = -2x + 6
\]

The point of intersection is (2,2). The solution to the system of equations is (2,2)

Check: (Substitute 2 for x and 2 for y)

\[
\begin{align*}
y &= 4x - 6 \\
2 &= 4(2) - 6 \\
2 &= 2
\end{align*}
\]

\[
\begin{align*}
2x + y &= 6 \\
2(2)+2 &= 6 \\
6 &= 6
\end{align*}
\]
14. Solve the following system of equations.
\[
\begin{align*}
1. \quad 2x - 3y + z &= 30 \\
2. \quad -2x + y - 3z &= -14 \\
3. \quad x - y + 2z &= 11
\end{align*}
\]

**Step 1:** Eliminate x from equations 1 & 2.
\[
\begin{align*}
2x - 3y + z &= 30 \\
-2x + y - 3z &= -14
\end{align*}
\]
\[
-2y - 2z = 16
\]

**Step 2:** Eliminate x from equations 2 & 3
\[
\begin{align*}
-2x + y - 3z &= -14 \\
2(x - y + 2z) &= 11(2)
\end{align*}
\]
\[
\begin{align*}
2x - 2y + 4z &= 22 \\
-2y + 4z &= 16
\end{align*}
\]
\[
-y + z = 8
\]

**Step 3:** Solve the system for equations 4 & 5. Eliminate y.
\[
\begin{align*}
-2y - 2z &= 16 \\
-2(-y + z) &= 8(-2)
\end{align*}
\]
\[
\begin{align*}
2y - 2z &= -16 \\
-4z &= 0
\end{align*}
\]
\[
z = 0
\]

**Step 4:** Substitute 0 for z into equation 4 or 5.
\[
\begin{align*}
-y + z &= 8 \\
-y + 0 &= 8 \\
y &= -8
\end{align*}
\]

**Step 5:** Substitute 0 for z, and -8 for y into one of the original equations.
\[
\begin{align*}
x - y + 2z &= 11 \\
x - (-8) + 2(0) &= 11 \\
x + 8 &= 11 \\
x + 8 - 8 &= 11 - 8 \\
x &= 3
\end{align*}
\]

Solution: (3, -8, 0)
15. The length of a flower bed is 1.5 times its width. The perimeter of the flower bed is 40 ft. Find the length of the flower bed. (2 Points)

In order to find the length, we will need to write a system of equations. We are given information about the length of the flower bed and the perimeter. We can write one equation about the length and one equation about the perimeter. We must know from previous knowledge that the perimeter formula is: \( P = 2l + 2W \) (2 times the length + 2 times the width equals perimeter.)

Let \( l = \text{length} \)  
Let \( w = \text{width} \)

\[ 2l + 2w = 40 \]  
Equation about perimeter. Substitute 40 for \( P \) since we know the perimeter is 40 ft.

\[ l = 1.5w \]  
Equation about width. The length is 1.5 times the width.

These two equations are set up to use the substitution method for solving!

\[ \begin{align*} 
L &= 1.5w \\
2l + 2w &= 40 \\
2(1.5w) + 2w &= 40 & \text{Substitute 1.5}w \text{ for } l \text{ into the equation.} \\
3w + 2w &= 40 & \text{Multiply } 2(1.5w) = 3w \\
5w &= 40 \\
\frac{5w}{5} &= \frac{40}{5} & \text{Divide by 5} \\
w &= 8 & \text{The width } = 8 \text{ ft.} \\
L &= 1.5(8) \\
L &= 12 \text{ ft} \\
\text{The length of the flower bed is 12 ft.} 
\end{align*} \]
16. The number of calories in a piece of cake is 30 more than the number of calories in a scoop of ice cream. The cake and ice cream together have 610 calories. How many calories are in each? (2 Points)

In this problem, we know information about how many calories total are in the cake and ice cream together. We also know that the cake has 30 more calories than the ice cream. Therefore, we can write a system of equations and solve in order to find out how many calories are in each.

Let \( x \) = # of calories in the cake
Let \( y \) = # of calories in the ice cream

\[
\begin{align*}
x + y &= 610 \\
x &= y + 30
\end{align*}
\]

The cake and ice cream together have 610 calories. Cake = 30 more calories than ice cream

Since the second equation is written as \( x = y + 30 \), we can use the substitution method to solve.

\[
\begin{align*}
x &= y + 30 \\
x + y &= 610 \\
y + 30 + y &= 610 \\
2y + 30 &= 610 \\
2y &= 580 \\
y &= 290
\end{align*}
\]

There are 320 calories in the cake and 290 calories in the ice cream.

Check:

\[
\begin{align*}
x + y &= 610 \\
320 + 290 &= 610 \\
610 &= 610
\end{align*}
\]

\[
\begin{align*}
x &= y + 30 \\
320 &= 290 + 30 \\
320 &= 320
\end{align*}
\]
17. The girl scouts are selling boxes of cookies. They pay $20 for a contract with the company and $6 a box for the cookies. They sell the cookies for $8 a box. (3 Points)

- Write a system of equations that represents this situation.
- How many boxes of cookies must the Girl Scouts sell in order to break even? Justify your answer.

We are given information about how much the girl scouts pay for the cookies and the selling price of the cookies. Therefore, one of our equations will be about the selling price, and one will be about how much they pay.

I am given a rate ($ per box) and I am given a constant ($20 contract). I am going to write my equation in slope intercept form.

Let \( x \) = the number of boxes of cookies

Let \( y \) = total amount

\[
Y = 6x + 20
\]

They pay $6 a box (rate/slope) and $20 for the contract (constant/y-intercept)

\[
Y = 8x
\]

They sell for $8 a box (rate/slope)

- The system of equations is:

\[
\begin{align*}
\text{y} &= 6x + 20 \\
\text{y} &= 8x
\end{align*}
\]

In order to find when the Girl Scouts will break even, we must find the when the two amounts will be the same. Therefore, we must find the solution to the system of equations! I’m going to use the substitution method.

\[
\begin{align*}
Y &= 6x + 20 \\
Y &= 8x \\
6x + 20 &= 8x \\
6x - 6x + 20 &= 8x - 6x \\
20 &= 2x \\
20 &= 2x \\
20 &= 2x \\
2 &= 2 \\
10 &= x
\end{align*}
\]

\[
\begin{align*}
y &= 8x \\
&= 8(10) \\
&= 80
\end{align*}
\]

The solution is (10,80)

- The Girl Scouts must sell 10 boxes to break even. Thereafter they will make a profit.

- Justify:

\[
\begin{align*}
Y &= 6x + 20 \\
80 &= 6(10)+20 \\
&= 8x \\
&= 8(10) \\
&= 80
\end{align*}
\]
18. During a conference lunches were ordered by 2 groups of people. See the table below. (4 Points)

<table>
<thead>
<tr>
<th></th>
<th>Number of Chicken Lunches</th>
<th>Number of Pasta Lunches</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>14</td>
<td>$176.50</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>19</td>
<td>$172.75</td>
</tr>
</tbody>
</table>

- Write a system of equations that represents this situation.
- What is the price of a chicken lunch?
- What is the price of a pasta lunch?
- Explain how you determined your answers.

We have information about chicken lunches and pasta lunches. My chart gives me information about two different groups.

I am going to write an equation to represent each group.

Let $x = \text{Price of chicken lunch}$  
Let $y = \text{Price of pasta lunch}$

$20x + 14y = 176.50$  
This equation represents the lunches ordered for group A

$15x + 19y = 172.75$  
This equation represents the lunches ordered for group B

- The system of equations for this situation is:

$20x + 14y = 176.50$  
$15x + 19y = 172.75$

We’ll need to solve the system of equations in order to find the price of each lunch. I am going to use linear combinations since both equations are written in standard form.

-3$(20x + 14y = 176.50)  
-60x -42y = -529.50$  
Multiply by -3 to make the x term -60

4$(15x + 19y = 172.75)  
60x +76y = 691$  
Combine like terms

$34y = 161.50$  
Divide both sides by 34

$34 34$

$Y = 4.75$  
The price of pasta lunch is $4.75$

$Y = 4.75$ (Substitute back into one of the equations to find x)

$20x + 14y = 176.50$
$20x + 14(4.75) = 176.50$
$20x +66.50 = 176.50$
$20x + 66.50 -66.50 = 176.50 - 66.50$

$20x = 110$
$20 20$

$x = 5.5$

The price of a pasta lunch is $4.75. The price of a chicken lunch is $5.50

I determined my answer by writing a system of equations I wrote an equation to represent each group and solved using linear combinations.